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ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

By THOMAS CRAIG, Ph.D., Fellow in Physics in the Johns Hopkins University, Baltimore, Md.

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III.

These considerations now enable us to determine p wholly in terms of the initial co-ordinates. To do this it is only necessary to write $x=x_0+u$, $z=z_0+w$, then from what has just been said we have at once

$$\sigma x=\sigma x_0,$$
$$\sigma z=\sigma z_0,$$

Substituting these in the expression for φ and this again in the equation giving the pressure and we obtain for the latter

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\left(\begin{matrix}\sigma z_0 & -\sigma z_0\\ a_1\varepsilon & +a_2\varepsilon\end{matrix}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x_0\right)+gz_0+gw\right\}+p_0$$

or as it may be written substituting for w its value

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\left(\begin{matrix}\sigma z_0 & -\sigma z_0\\ a_1\varepsilon & +a_2\varepsilon\end{matrix}\right)+g\sigma\left(\begin{matrix}\sigma z_0 & -\sigma z_0\\ a_1\varepsilon & -a_2\varepsilon\end{matrix}\right)\right\}\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right)+p_0+\rho gz_0$$

Now for the determination of the constants, we observe first: that particles of the fluid originally on the bottom of the canal, must necessarily remain there during the motion; second, for particles of the fluid whose z co-ordidates are equal to zero, *i.e.*, for particles on the surface of the fluid at rest, p must be a constant $=p_0$. Let h denote the depth of the canal and the first of these conditions is evidently reached by making $w=o$ for those particles for which $z=h$. This gives us then

$$w=\sigma\left(\begin{matrix}\sigma h & -\sigma h\\ a_1\varepsilon & -a_2\varepsilon\end{matrix}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right)=o$$

and consequently

$$\begin{matrix}\sigma h & -\sigma h\\ a_1\varepsilon & -a_2\varepsilon\end{matrix}=o$$

from which follows,

$$\frac{a_1}{a_2}=\frac{\begin{matrix}\varepsilon & -\sigma h\end{matrix}}{\begin{matrix}\sigma h & \varepsilon\end{matrix}}$$

Again, make $z=o$ and $p=p_0$; this gives obviously,

$$\left(\frac{2\pi}{\tau}\right)^2(a_1+a_2)+g\sigma(a_1-a_2)=o$$

from which

$$\left(\frac{2\pi}{\tau}\right)^2=-g\sigma\frac{a_1-a_2}{a_1+a_2}=g\sigma\frac{\begin{matrix}\sigma h & -\sigma h\\ \varepsilon & -\varepsilon\end{matrix}}{\begin{matrix}\sigma h & -\sigma h\\ \varepsilon & +\varepsilon\end{matrix}}$$

Now making for brevity

$$\sigma a_2 \varepsilon^{-\sigma h} = -a$$

And our expressions for u and v become,

$$u = -a\left(\varepsilon^{-\sigma(h-z)} + \varepsilon^{\sigma(h-z)}\right)\cos.\left(\frac{2\pi}{\tau}t + \sigma x\right)$$

$$v = -a\left(\varepsilon^{-\sigma(h-z)} - \varepsilon^{\sigma(h-z)}\right)\sin.\left(\frac{2\pi}{\tau}t + \sigma x\right)$$

By introducing the wave length now we can determine the constant σ. Reverting to the expression previously given for φ, and for convenience retain the *two* constants a_1 and a_2—this was

$$\varphi = \left(a_1\varepsilon^{\sigma z} + a_2\varepsilon^{-\sigma z}\right)\sin.\left(\frac{2\pi}{\tau}t + \sigma x\right).$$

Let l denote the wave length; we know that φ will remain unchanged by writing for x the quantity $x+l$, as this simply has the effect of transferring the origin of co-ordinates from one end of the wave to the other. This substitution gives

$$\varphi = \left(a_1\varepsilon^{\sigma z} + a_2\varepsilon^{-\sigma z}\right)\sin.\left(\frac{2\pi}{\tau}t + \sigma[x+l]\right).$$

In order that φ remain unchanged, we must clearly have

$$\sigma = \frac{2\pi}{l}.$$

If we call ω the velocity of each particle, we have also

$$l = \tau\omega, \text{ or } \tau = \frac{l}{\omega}$$

Substituting those in the expression obtained above for p, and it becomes

$$p = p_0 + \rho g z_0 + \rho\left\{g\sigma\frac{\varepsilon^{\sigma h} - \varepsilon^{-\sigma h}}{\varepsilon^{\sigma h} + \varepsilon^{-\sigma h}}\left(a_2\frac{\varepsilon^{-\sigma h}}{\sigma h}\varepsilon^{\sigma z_0} + a_2\varepsilon^{-\sigma z_0}\right)\right.$$

$$\left. + g\sigma\left(a_2\frac{\varepsilon^{-\sigma h}}{\sigma h}\varepsilon^{\sigma z_0} - a_2\varepsilon^{-\sigma z_0}\right)\right\}\sin.\frac{2\pi}{\rho}(\omega t + x_0)$$

which reduces easily to

$$p = p_0 + \rho g z^0 - z\rho g a\left\{\frac{\varepsilon^{\frac{2\pi}{l}z_0} - \varepsilon^{-\frac{2\pi}{l}z_0}}{\varepsilon^{\frac{2\pi}{l}h} + \varepsilon^{-\frac{2\pi}{l}h}}\right\}\sin.\frac{2\pi}{l}(\omega t + \varepsilon)$$

The same substitutions give us for φ the value

$$\varphi = -\frac{al}{2\pi}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} - \varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\sin.\frac{2\pi}{l}(\omega t + x)$$

and in like manner we can obtain for u and v the values

$$u = -a\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} + \varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\cos.\frac{2\pi}{l}(\omega t + x),$$

$$v = -a\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} - \varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\sin.\frac{2\pi}{l}(\omega t + x).$$

The value of ω is easily obtained.

$$\left(\frac{2\pi}{\tau}\right)^2 = \left(\frac{2\pi\omega}{l}\right)^2 = g\sigma\frac{\varepsilon^{\sigma h} - \varepsilon^{-\sigma h}}{\varepsilon^{\sigma h} + \varepsilon^{-\sigma h}} = g\frac{2\pi}{l}\frac{\varepsilon^{\frac{2\pi}{l}h} - \varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h} + \varepsilon^{-\frac{2\pi}{l}h}}$$

from this is readily obtained

$$\omega = \sqrt{\frac{gl}{2\pi}\cdot\frac{\varepsilon^{\frac{2\pi}{l}h} - \varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h} + \varepsilon^{-\frac{2\pi}{l}h}}}$$

In the discussion of these values for φ, u, v and p lies the whole theory of the motion of plane waves in a perfect fluid. We will now proceed to an examination of these quantities. Denote by z' the vertical ordinate at the time t of a particle on the wave surface whose other co-ordinates are x and y.

$$v = \frac{dz'}{dt} = -a\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} - \varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\sin.\frac{2\pi}{l}(\omega t + x)$$

integrating

$$z' = \frac{a\rho}{2\pi\omega}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} + \varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\cos.\frac{2\pi}{l}(\omega t + x).$$

Differentiating this expression with respect to x and we obtain.

$$\frac{dx'}{dx}=-\frac{a}{\omega}\left\{-\underset{\varepsilon}{\frac{z\pi}{l}(h-z)}\ \underset{\varepsilon}{-}\ \frac{z\pi}{l}(h-z)\right\}\sin.\ \underset{(\omega t+x)}{\frac{2\pi}{l}}$$

This vanishes for the values

$$\omega t+x=o$$
$$\omega t+x=\tfrac{l}{2}$$
$$\omega t+x=l,$$

which are the values of $\omega t+x$ and for the points of maxima and minima of the longitudinal section of the wave. Differentiating again

$$\frac{d^2z'}{dx^2}=-\frac{a\rho}{2\pi\omega}\left\{-\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\ \underset{\varepsilon}{-}\ \frac{2\pi}{l}(h-z)\right\}\cos.\ \underset{(\omega t+x)}{\frac{2\pi}{l}}$$

This vanishes for $\omega t+x=\frac{l}{4}$, and $\frac{3l}{4}$.

Consequently there are points of contra-flexure at $\frac{1}{4}$ and $\frac{3}{4}$ of the wave length. It is obvious, from these considerations, what the curve is.

It has before been remarked that u and v satisfy the equation of an ellipse; this is now

$$\frac{u^2}{a^2\left\{-\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\ \underset{\varepsilon}{+}\ \frac{2\pi}{l}(h-z)\right\}^2}+\frac{u^2}{a^2\left\{-\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\ \underset{\varepsilon}{-}\ \frac{2\pi}{l}(h-z)\right\}^2}=1$$

The plane of the ellipse is vertical and its longer axis is in the direction of the motion of the wave. Suppose now the particle under consideration to lie very near the surface of the wave—that is, z is very small as compared with h; then the terms containing $\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}$ may obviously be discarded, and the only other terms which remain in the expressions for the semi-axes of the ellipse will depend on

$$\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\quad \frac{2\pi}{l}h-\frac{2\pi}{l}z\quad -\frac{2\pi'}{l}z.$$
$$=\varepsilon\ .\ \varepsilon\ =\ \mathrm{A}\varepsilon.$$

The equation of the ellipse thus becomes

$$\frac{u^2}{\mathrm{A}^2-\underset{\varepsilon}{\frac{4\pi}{l}z}}+\frac{v^2}{\mathrm{A}^2-\underset{\varepsilon}{\frac{4\pi}{l}z}}=1$$

The equation of a circle whose radius is

$$\mathrm{A}\underset{\varepsilon}{\frac{2\pi}{l}z}=\mathrm{R}$$

Of course the same result would be obtained by supposing the depth of the fluid infinite. Thus for particles near the surface of a body of water of finite depth—or for particles anywhere within the mass of a body of water of infinite depth—the motion is in a vertical circle whose radius is given above. Suppose again that the wave has an appreciable length—say $l=h$; then for particles very near the surface the semi-axes become very nearly.

$$\underset{\varepsilon}{\frac{2\pi}{}}\ \underset{\varepsilon}{+}\ \frac{-2\pi}{}, \text{ and } \underset{\varepsilon}{\frac{2\pi}{}}\ \underset{\varepsilon}{-}\ \frac{-2\pi}{}$$

or the path of the particle is nearly circular—the ratio between these quantities being nearly 1.000,007.

The lengths of the axis continuously decrease as z increases. This is obvious in the case of the vertical axis given by

$$2a\left\{\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\ \underset{\varepsilon}{-}\ \frac{2\pi}{l}(h-z)\right\}$$

for as z becomes larger the exponents in this quantity become smaller, thus causing the first term in the brackets to diminish as the second increases, and consequently making the total value of the quantity diminish rapidly. Take now the horizontal axis denoted by

$$2a\left\{\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\ \underset{\varepsilon}{+}\ \frac{-2\pi}{l}(h-z)\right\}=\beta$$

differentiating this with respect to z and we have

$$\frac{d\beta}{dz}=2a\frac{2\pi}{l}\left\{-\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\ \underset{\varepsilon}{-}\ \frac{2\pi}{l}(h-z)\right\}$$

For $z<h$ the second of these terms is always greater than the first and, consequently, $\frac{d\beta}{dz}$ is negative; but the increment dz is supposed positive, consequently $d\beta$ is negative, or the axis decreases as z increases, *i.e.*, as we pass from the surface of the fluid. For $z=h$ this axis becomes $=2a$, and the vertical axis vanishes. That is, for particles of

water at the bottom of the canal there is only a motion of translation backwards and forwards in lines of length $=2a$. For particles at the surface of the fluid and for $h=l$ the horizontal axis is nearly $=535.5a$, or the ratio between the lengths of the horizontal axis at top and bottom of the fluid is nearly 267.7.

Referring now to the values of u and w near the surface, we have

$$\begin{cases} u=-\mathrm{A}\varepsilon^{-\frac{2\pi}{l}z}\cos.\frac{2\pi}{l}(\omega t+x) \\ w=\mathrm{A}\varepsilon^{-\frac{2\pi}{l}z}\sin.\frac{2\pi}{l}(\omega t+x) \end{cases}$$

Differentiating these for t, squaring and adding the results, and we obtain the expression

$$\left(\frac{du}{dt}\right)^2+\left(\frac{dw}{dt}\right)^2=\left(\frac{2\pi\omega}{l}\mathrm{R}\right)^2$$

The quantity on the left hand side of this equation gives the square of the velocity of the fluid particle in its circular path; this, as we see from the second member of the equation, is independent of the time, and is directly proportional to the radius of the circle; but the radius depends upon the quantity $\varepsilon^{-\frac{2\pi}{l}z}$ for its value, and this increases as z decreases—therefore, for particles near the surface of shallow water, we have the velocity x varies inversely as their depth. From this it is evident that the water at the top of the wave moves most rapidly forward, while that at the bottom moves most rapidly backward. In the expression for ω discarding the terms $-\varepsilon^{-\frac{2\pi}{l}h}$ we have for the velocity of translation of particles near the surface of water of finite depth, or anywhere within the mass of a body of water of infinite depth

$$\omega=\sqrt{\frac{gl}{2\pi}}.$$

Substituting for ω its value of $\frac{l}{\tau}$ and we find for τ the value

$$\sqrt{\frac{2\pi l}{g}}.$$

Calling t' the time of oscillation of a simple pendulum of length l, we have

$$\tau=\sqrt{\frac{2}{\pi}}t'.$$

From the above value of ω we see that the velocity of transmission of the wave varies as the square root of the length. In all cases, indeed, the velocity is nearly as the square root of the length, for the factor

$$\frac{\varepsilon^{\frac{2\pi}{l}h}-\varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h}+\varepsilon^{-\frac{2\pi}{l}h}}$$

is nearly equal to unity. In the case of very shallow water, the velocity diminishes considerably—as the quantity just written decreases rapidly with h—vanishing as is obvious for $h=o$.

So far we have confined ourselves to a single wave, that is, to a single value of φ satisfying the equation $\Delta^2\varphi=o$. But we have seen that if there are several values of φ each satisfying this equation, that collectively they satisfy the equation

$$\Delta^2\Sigma\varphi=o.$$

In the case when the wave lengths are the same but the phases different, we can easily find the result of adding together the waves given by the functions φ, φ_1, $\ldots$ φ_i.

The value that we have already obtained for φ may be written

$$-\frac{a_0}{\sigma}\left\{\varepsilon^{\sigma(h-z)}-\varepsilon^{-\sigma(h-z)}\right\}\sin.\sigma(\omega t+x+a_0)$$

where it is to be understood that $a_0=o$ and is merely introduced for future symmetry. Any other function φ_i which satisfies the equation $\Delta^2\varphi=o$, may be written under the above conditions

$$\varphi_i=-\frac{a_i}{\sigma}\left(\varepsilon^{\sigma(h-z)}-\varepsilon^{-\sigma(h-z)}\right)\sin.\sigma(\omega t+x+a_i)$$

And it is not difficult to see that a summation of these functions will give us

$$\Sigma\varphi=-\frac{\mathrm{A}}{\sigma}\left\{\varepsilon^{\sigma(h-z)}-\varepsilon^{-\sigma(h-z)}\right\}\sin.\sigma(\omega t+x+\Psi)$$

When

$$A^2 = \sum_{i=o} a_i^2 + 2\sum_{j=o} a_j \sum_{k=j+1} a_k \cos.\sigma\,(a_{k-1}-a_k)$$

and

$$\tan.\Psi = \frac{\sum_{i=1} a_i \sin.\ \sigma a_i}{\sum_{j=o} a_j \cos.\ \sigma a_j}$$

or inversely

$$\Psi = \frac{1}{\sigma}\tan.\ \frac{-1\ \sum_{i=1} a_i \sin.\ \sigma\, a_i}{\sum_{j=0} a_j \cos.\ \sigma a_j}$$

If any of the quantities σa, &c., $=\pi$ or $(2n+1)\pi$ a change takes place in the summation. Suppose $\sigma a_i = \pi$ then φ_i becomes $-\varphi_i$ and is subtracted instead of added to the other functions; but if $\sigma a_i = (2n+1)\pi$ we have since $\sigma = \frac{2\pi}{l}$,

$$a_i = (2n+1)\frac{l}{2}.$$

That is if the difference of phase is an odd multiple of half the wave length, the corresponding wave function is to be subtracted instead of added to the others in finding the resultant of the system.

Suppose now that we have two waves of the same length and amplitude, but of different phases and moving in opposite directions; the wave functions are obviously

$$\varphi = -\frac{a}{\sigma}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}\right\}\sin.\ \sigma\ (\omega t + x)$$

$$\varphi_1 = -\frac{a}{\sigma}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}\right\}\sin.\ \sigma\ (\omega t - x + a)$$

adding we have

$$\varphi+\varphi_1 = -\frac{a}{\sigma}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}\right\}\sin.\ \sigma\ (\omega t + x) + \sin.\ \sigma\ (\omega t - x + a)$$

expanding the trigonometric factor and reducing by aid of the relations

$$\cos.\ a = 2\cos.^2\frac{a}{2} - 1,\quad \sin.\ a = l\sin.\frac{a}{2}\cos.\frac{a}{2},$$

we readily find this expression to become

$$\varphi+\varphi_1 = -\frac{2a}{\sigma}\left\{\begin{matrix}(\sigma h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}\right\}\sin.\ \sigma\ (\omega t + \frac{a}{2})\ \cos.\ \sigma\ (x-\frac{a}{2}).$$

From this we obtain by differentiation the values of the displacements

$$u = 2a\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}\right\}\sin.\ \sigma\left(x-\frac{a}{2}\right)\ \sin.\ \sigma\left(\omega t+\frac{a}{2}\right)$$

$$w = 2a\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & -\ \varepsilon\end{matrix}\right\}\sin.\ \sigma\left(\omega t+\frac{a}{2}\right)\ \cos.\ \sigma\left(x-\frac{a}{2}\right).$$

Dividing the first of these by the second we have

$$\frac{u}{w} = \frac{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}}{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & -\ \varepsilon\end{matrix}}\tan.\sigma\left\{x-\frac{a}{2}\right\};$$

this ratio is independent of the time, consequently each particle moves in a straight line the inclination of which varies with x and z. Since also this ratio is the same at any given point, no matter what be the time, the wave is a standing wave or has no progressive motion.

Hence if there exist in the liquid two waves having the same length and amplitude but moving in opposite directions the result is a single standing wave in which the particles move constantly in right lines whose inclinations to the axes vary with x and z.

Reverting now to our values of u and w, suppose that $u=o$, that is, that there be no horizontal motion, this gives us

$$\cos.\ \frac{2\pi}{l}(\omega t+)=o$$

or $$\omega t + x = \frac{l}{4}\text{ or }\frac{3l}{4}\text{ or }\frac{(2n+1)l}{4}.$$

That is, there is no horizontal motion at the nodes of the wave. The greatest horizontal motion evidently corresponds to

$$\omega t + x = o, \frac{l}{2}, \text{ or } l$$

or the greatest horizontal motion is at the highest point of the crest and the lowest point of the trough of the wave—and evidently the motions at these points are in opposite directions—which we have seen before from other considerations.

In like manner by making $w=o$ we find that at the top and bottom of the wave in there is no vertical motion. Also, that the greatest vertical motion is at the nodes of the wave.

Similar results are obtained by exam-

ining the equations a. There is no horizontal motion at the points when $x-\frac{a}{2}=\frac{l}{4}$ or $\frac{3l}{4}$ but there is a maximum of vertical motion; also, there is no vertical motion at the points where $x-\frac{a}{2}=o, \frac{l}{2}$, or l, but there is the greatest horizontal motion.

Airy has shown in his treatise on "Tides and Waves," that if the channel is of variable depth or width, that waves of the nature just described, that is, waves caused by the simple oscillation of the particles, could not exist by themselves, but require for their existence the action of certain exterior forces into the nature of which it is not necessary here to go. Without going into a mathematical discussion of the reflection of waves, I will merely state that after impinging upon a wall the particles of the wave move up and down the surface through a distance equal to twice their previous vertical displacement, and the same with particles at a distance of half a wave length from the wall; particles at a distance from the wall of one quarter a wave length, merely vibrate in a horizontal direction. When a series of waves enters shallow water, the period remains the same, but the velocity and wave length diminish; the front of the wave becomes steeper than the back, and continues to become more and more abrupt until the top of the wave curls over the front and the wave breaks in surf on the beach.

§ 4.

CYLINDRICAL WAVES.

If we throw a pebble into a body of water, or if we simply bring a solid body in contact with the water at one point we know that a series of waves is generated which are circular in form, concentric and having their center at the point where the disturbance takes place. The waves thus generated are called cylindrical waves, and the line passing through the center of these circles and normal to the surface of the fluid is called the wave-axis, and evidently is the geometrical axis of the concentric cylinders.

In the case of such waves as this it is evidently not admissible to assume the displacement in any direction as equal to zero, there will clearly be motion in the direction of all these axes. Our axis of z will be assumed as having the same direction as in the foregoing section, and the axes of X and Y will lie in the surface of the fluid at rest. Our equation of continuity will have the general form

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=o$$

the displacements being of course given by

$$u=\frac{d\varphi}{dx},\quad v=\frac{d\varphi}{dy},\quad w=\frac{d\varphi}{dz}.$$

The same remarks that were previously made concerning the form of φ will hold here, the waves being supposed to emanate from the wave axis, oz we can write for φ the equation

$$\varphi=\frac{\pm\sigma z}{\varepsilon} f(x, y) \begin{matrix}\sin\\ \cos\end{matrix}\left\{\frac{2\pi}{\tau}t\right\}$$

when τ as before denotes the periodic time. If the wave axis be taken as the axis of z we have, r denoting the distance from this axis to any point in a plane parallel to the plane of $x\ y$,

$$r^2=x^2+y^2$$

and we may with φ in the form,

$$\varphi=\frac{\pm\sigma z}{\varepsilon} f(r) \begin{matrix}\sin\\ \cos\end{matrix}\left\{\frac{2\pi}{\tau}t\right\}$$

We must now, as before, determine the form of f. Substitute this value of φ in the equation of continuity, and it is easily found to reduce to the form

$$\frac{d^2f}{dr^2}+\frac{1}{r}\frac{df}{dr}+\sigma^2 f=o$$

Transforming this by the substitution

$$r=\frac{s}{\sigma}$$

we obtain a known form

$$\frac{d^2f}{ds^2}+\frac{1}{s}\frac{df}{ds}+f=o.$$

This is a particular case of the more general equation,

$$\frac{d^2f}{ds^2}+\frac{1}{s}\frac{df}{ds}+\left(1-\frac{n^2}{s^2}\right)f=o$$

of which a particular solution is the Bessel's function $J_n(s)$ given by

$$J_n(s)= \frac{s_n}{2^n n!}\left\{1-\frac{s^2}{2(2n+2)}+\frac{s^4}{2.4(2n+2)(2n+4)}-\frac{s^6}{2.4.6(2n+2)(2n+4)(2n+6)}+\ldots.\right\}$$

For our case $n=o$, and the function $J_o(s)$ is a particular solution, viz.:

$$J_o(s)=\left\{1-\frac{s^2}{2^2}+\frac{s^4}{2^2.4^2}-\frac{s^6}{2^2.4^2.6^2}+\ldots..\right\}$$

This is easily obtained directly, calling f_o the particular solution sought assume

$$f_o=a+a_1s+a_2s^2+a_3s^3+a_4s^4+\ldots.$$

Substituting in the differential equation, and we have

$$o=a_1s^{-1}+(a+4a_2)+(a_1+9a_3)s+(a_2+16a_4)s^2+\ldots.$$

from which we have

$$a_1=a_3=a_5=\ldots\ldots\ a_{2i+1}=o$$

$$a_2=-\frac{a}{4},\ a_4=\frac{a}{64}=\frac{a}{2^2.4.^2},\ \text{\&c.}\ldots..$$

when i is of course any positive integral. This gives us then for our particular solution

$$f_o=J_o(s)=a\left\{1-\frac{s^2}{2^2}-\frac{s^4}{2^2.4^2}+\frac{s^6}{2^2.4^2.6^2}+\ldots\right\}$$

Designate now by $Y_o(s)$ the other particular solution of the differential equation, and for brevity write simply Y_o and J_o instead of $Y_o(s)$ and $J_o(s)$. Let now Σ denote a function of s, then it is well known that we can write

$$Y_o=J_o\Sigma$$

Substituting this in the differential equation and it becomes

$$\left(\frac{1}{s}+\frac{2}{J_o}\frac{dJ_o}{ds}\right)\frac{d\Sigma}{ds}+\frac{d^2\Sigma}{ds^2}=o$$

Dividing this by $\frac{d\Sigma}{ds}$ and integrating, gives

$$\log. s+2\log. J_o+\log.\frac{d\Sigma}{ds}=\text{const.}$$

or assuming the const. $=o$, and passing to exponentials

$$sJ_o^2\frac{d\Sigma}{ds}=1$$

from which

$$\frac{d\Sigma}{ds}=\frac{1}{sJ_o^2}$$

and

$$\Sigma=\int\frac{ds}{sJ_o^2}$$

and, by substitution in the equation giving Y_o,

$$Y_o=J_o\int\frac{ds}{sJ_o^2}.$$

Now from the value of J_o it is clear that the expansion of $\frac{1}{J_o^2}$ can only contain even powers of s and we can thus write

$$\frac{1}{sJ_o^2}=\frac{1}{s}\left\{1+As^2+Bs^4+Cs^6+\right\};$$

multiplying by ds and integrating gives

$$Y_o=J_o\log. s+J_o\left\{A\frac{s^2}{2}+B\frac{s^4}{4}+C\frac{s^6}{6}+\right\}$$

or as it may be written for brevity

$$Y_o=J_o\log. s+E_o$$

The quantity E_o is the product of two infinite series each of which contains only positive integral powers of s and, consequently, according to a principle in the theory of the Bessel's functions can be developed in a series of these functions and, moreover, as all the powers in J_o and the other factor of E_o are even, only the even Bessel's functions will appear in the development thus

$$Y_o=J_o\log. s+aJ^0+bJ^2+cJ^4+\ldots.$$

The co-efficients a, b, c have to be determined.

Take again the differential equation

$$\frac{d^2f}{ds^2}+\frac{1}{s}\frac{df}{ds}+f=o$$

and perform the operation

$$\frac{d^2}{ds^2}+\frac{1}{s}\frac{d}{ds}+1$$

on the quantity $J_o\log. s$ and we find

$$\left\{\frac{d^2}{ds^2}+\frac{1}{s}\frac{d}{ds}+1\right\}J_o\log. s=\frac{2}{s}\frac{dJ_o}{ds}.$$

Represent the operator for brevity by Δ, then this is

$$\Delta J_o\log. s=\frac{2}{s}\frac{dJ^o}{ds}$$

We have now from the general differential equation affording Bessel's functions

$$\Delta J_n=\frac{n^2}{s^2}J_n$$

and for $n=o$

$$\Delta J_o=o$$

and for $n>o$

$$\Delta J_n = \frac{n}{2s}\left\{ J_{n-1} + J_{n+1} \right\}$$

according to a known relation connecting these three consecutive functions. And so we have, finally

$$\Delta J_0 \log. s = -\frac{2}{s} J_1$$

Now from the above value of Y_0 we have

$$\Delta Y_0 = \Delta(J_0 \log. s) + a\Delta J_0 + b\Delta J_2 + c\Delta J_4 + \ldots.$$

And by aid of the transformations just given

$$\Delta Y_0 = -\frac{2}{s} J_1 + \frac{b}{s}(J_1 + J_3) + \frac{2C}{s}(J_3 + J_5) + \frac{3d}{s}(J_5 + J_7) \times \ldots$$

Now Y_0 being a particular solution of the differential equation $\Delta f = o$, we must have $\Delta Y_0 = o$; this enables us to find

$$b = -2c = 3d = -4e = 5g = \&c \ldots\ldots\ldots = 2$$

and by substitution

$$Y_0 = J_0 \log. s + aJ_0 + 2[J_2 - \tfrac{1}{2}J_4 + \tfrac{1}{3}J_6 - \tfrac{1}{4}J_8 + \ldots.]$$

The complete solution of the differential equation $\Delta f = o$, *i.e.*,

$$\frac{d^2 f}{ds^2} + \frac{1}{s}\frac{df}{ds} + f = o$$

is now given by

$$f = AJ_0 + BY_0$$

or

$$= (A + B \log. s)\, J_0 + BE_0$$

It may be verified without much difficulty that the quantity E_0 is of the form

$$E_0 = \left\{ \frac{s^2}{2^2} - \frac{1+\frac{1}{2}}{2^2.4^2} s^4 + \frac{1+\frac{1}{2}+\frac{1}{3}}{2^2.4^2.6^2} s^6 + \ldots.. \right\}$$

The quantities J_0 and Y_0 expressed in the form of definite integrals are—vide Boole's Diff. Equas.

$$J_0 = \frac{1}{\pi}\int_0^{\pi} \cos(s \sin \omega)\, d\omega$$

$$Y_0 = \frac{1}{\pi}\int_0^{\pi} \cos(s \sin \omega) \log(4 s \cos^2 \omega)\, d\omega$$

and we have for f by substitution

$$f = \frac{1}{\pi}\int_0^{\pi} \cos(s \sin \omega)\,(A + B \log (4s \cos^2 \omega))\, d\omega$$

or as this may be written

$$f = \int_0^{\pi} \cos(s \sin \omega)\,(C + D \log (s \cos^2 \omega))\, d\omega$$

when

$$C = \frac{A + B \log. 4}{\pi}, \quad D = \frac{B}{\pi}.$$

Before going on to the application of these results to the problem in hand, we will investigate the change produced in the quantities J_0 and Y_0 by allowing s to become very great.

Instead of f in the differential equation $\Delta f = o$ write $f\sqrt{s}$ this equation thus becomes

$$\frac{d^2(f\sqrt{s})}{ds^2} + \left(1 + \frac{1}{4s^2}\right) f\sqrt{s} = o$$

and this for s very large is simply

$$\frac{d^2(f\sqrt{s})}{ds^2} + f\sqrt{s} = o.$$

This equation gives on integration

$$f\sqrt{s} = a \cos s + b \sin s$$

or

$$f = \frac{a \cos s + b \sin s}{\sqrt{s}}$$

when a and b are of course constants. We have then obviously from this

$$J_0 = \frac{a \cos s + \beta \sin s}{\sqrt{s}}$$

$$Y_0 = \frac{a' \cos s + \beta' \sin s}{\sqrt{s}}$$

from which we can see that for infinitely great values of s the functions J_0 and Y_0 will vanish.

Now by substituting for s its value of σr we can, by taking as the argument of the functions thus obtained the quantity $-\frac{s^2}{2^2}$, or $-\frac{\sigma^2 r^2}{4}$ with f in the form

$$f = \left\{ 1 + \sum_{i=1}^{i=s} \frac{\theta^i}{(i!)^2} \right\} (A + B \log. 2\sqrt{-\theta}) - B \sum_{i=1}^{i=x} \frac{\theta^i}{(i!)^2} \cdot \sum_{j=1}^{j=i} \frac{1}{j}$$

For convenience of reference hereafter we shall write this in the form,

$$f = A\Pi(\theta) + B\Omega(\theta).$$

Substituting this value of f in the expression for φ we obtain,

$$\varphi = \varepsilon^{\pm \sigma z}[A\Pi(\theta) + B\Omega(\theta)]\left\{ \sin \frac{2\pi}{\tau} t + \cos \frac{2\pi}{\tau} t \right\}$$

or expanding this and writing instead of

A and B, the quantities a_1, b_1, a_2, b_2, α_1, β_1, α_2, β_2, we have

$$\varphi = \pi \sin \frac{2\pi}{\tau} t \left(\begin{matrix} a_1\varepsilon + \alpha_1\varepsilon \\ \sigma z \end{matrix} \quad \begin{matrix} \\ -\sigma z \end{matrix} \right) + \pi \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ +\alpha_2\varepsilon \end{matrix} \right) + \Omega \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ b_1\varepsilon + \beta_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \Omega \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ b_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ +\beta_2\varepsilon \end{matrix} \right)$$

This gives us for u, v, w the following values;

$$u = \left\{ \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_1\varepsilon + \alpha_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a^2\varepsilon + a_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) \right\} \frac{d\pi}{dx} + \left\{ \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ b_2\varepsilon + \beta_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ b_2\varepsilon + \beta_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) \right\} \frac{d\Omega}{dx},$$

$$v = \left\{ \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_1\varepsilon + \alpha_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_2\varepsilon + a_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) \right\} \frac{d\pi}{dy} + \left\{ \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ b_1\varepsilon + \beta_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ b_2\varepsilon + \beta_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) \right\} \frac{d\Omega}{dy},$$

$$w = \pi \sin \frac{2\pi}{\tau} t\, \sigma \left(\begin{matrix} \sigma z \\ a_1\varepsilon - a_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \pi \cos \frac{2\pi}{\tau} t\, \sigma \left(\begin{matrix} \sigma z \\ a_2\varepsilon - a_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \Omega \sin \frac{2\pi}{\tau} t\, \sigma \left(\begin{matrix} \sigma z \\ b_1\varepsilon - \beta_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + \Omega \cos \frac{2\pi}{\tau} t\, \sigma \left(\begin{matrix} \sigma z \\ b_2\varepsilon - \beta_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right)$$

The expression for the fluid pressure is obtained here in the same manner as in the case of plane waves, and is

$$p = \rho \left\{ \left(\frac{2\pi}{\tau} \right)^2 \varphi + gz \right\} + \text{const.}$$

Writing as before $z = z_0 + w$, remembering that σ and w are quantities of the first order of magnitude, and so discarding terms containing σw or higher orders, we have

$$p = p_0 + \rho y z_0 + \rho \left\{ \pi \sin \frac{2\pi}{\tau} t \left(\frac{2\pi}{\tau} \right)^2 \left(\begin{matrix} \sigma z_0 \\ a_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z_0 \\ +a\varepsilon \end{matrix} \right) + y\sigma \left(\begin{matrix} \sigma z_0 \\ a_1\varepsilon - \end{matrix} \quad \begin{matrix} -\sigma z_0 \\ a_1\varepsilon \end{matrix} \right) \right\} + \pi \cos \frac{2\pi}{\tau} t \left\{ \left(\frac{2\pi}{\tau} \right)^2 \left(\begin{matrix} \sigma z_0 \\ a_2\varepsilon + \alpha\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z_0 \\ \end{matrix} \right) + y\sigma \left(\begin{matrix} \sigma z \\ a_2\varepsilon - a_2\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z_0 \\ \end{matrix} \right) \right\} + \Omega \sin \frac{2\pi}{\tau} \tau \left\{ \left(\frac{2\pi}{\tau} \right)^2 \left(\begin{matrix} \sigma z_0 \\ b_1\varepsilon + \beta_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z_0 \\ \end{matrix} \right) + y\sigma \left(\begin{matrix} \sigma z_0 \\ b_1\varepsilon - \beta_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z_0 \\ \end{matrix} \right) \right\} + \Omega \cos \frac{2\pi}{\tau} t \left\{ \left(\frac{2\pi}{\tau} \right)^2 \left(\begin{matrix} \sigma z_0 \\ b_1\varepsilon + \beta_2\varepsilon \end{matrix} \quad \begin{matrix} \sigma - z_0 \\ \end{matrix} \right) + y\sigma \left(\begin{matrix} \sigma z_0 \\ b_2\varepsilon - \beta_2\varepsilon \end{matrix} \quad \begin{matrix} \sigma - z_0 \\ \end{matrix} \right) \right\}$$

For particles on the surface of the fluid at rest we have, of course, $z_0 = o$ and $p = p_0$. This gives us

$$\pi \sin. \frac{2\pi}{\tau} t \left\{ \left(\frac{2\pi}{\tau} \right)^2 (a_1 + a_1) + g\sigma(a_1 - a_1) \right\} + \pi \cos \frac{2\pi}{\tau} t \left\{ \left(\frac{2\pi}{\tau} \right)^2 (a_2 + a_2) + g\sigma(a_2 - a_2) \right\} + \Omega \sin \frac{2\pi}{\tau} t \left\{ \left(\frac{2\pi}{\tau} \right)^2 [b_1 + \beta_1] + g\sigma \left(b_1 - \beta_1 \right) \right\} + \Omega \cos \frac{2\pi}{\tau} t \left\{ \left(\frac{2\pi}{\tau} \right)^2 \left\{ b_2 + \beta_2 \right\} + g\sigma \left(b_2 - \beta_2 \right) \right\} = o$$

In order that this may be satisfied we must have obviously

$$\frac{a_1 - a_1}{a_1 + a_1} = \frac{a_2 - a_2}{a_2 + a_2} = \frac{b_1 - \beta_1}{b_1 + \beta_1} = \frac{b_2 - \beta_2}{b_2 + \beta_2} = \frac{\left(\frac{2\pi}{\tau} \right)^2}{g\sigma}$$

from which we obtain,

$$\frac{a_1}{a_1} = \frac{a_2}{a_2} = \frac{b_1}{\beta_1} = \frac{b_2}{\beta_2}$$

We can now write

$$w = \sigma\pi \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_1\varepsilon - a_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + c_1\sigma\pi \cos \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_1\varepsilon - a_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + c_2\sigma\Omega \sin \frac{2\pi}{\tau} t \left(\begin{matrix} \sigma z \\ a_1\varepsilon - a_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right) + c_2\sigma\Omega \cos \frac{2\pi}{\tau} t \left(\begin{matrix} +\sigma z \\ a_1\varepsilon - a_1\varepsilon \end{matrix} \quad \begin{matrix} -\sigma z \\ \end{matrix} \right)$$

when the meaning of the constants

c_1, c_2, c_3 is obvious. Introducing now the condition that $w=o$ for $h=o$ we have as in plane waves

$$\frac{a_1}{a_1} = \frac{\overset{-\sigma h}{\varepsilon}}{\underset{\varepsilon}{\sigma h}};$$

This gives us again

$$\left(\frac{2\pi}{\tau}\right)^2 = g\sigma \frac{\overset{\sigma h}{\varepsilon} - \overset{-\sigma h}{\varepsilon}}{\underset{\varepsilon}{\sigma h} - \underset{\varepsilon}{\sigma h}}.$$

We have now for φ the equation,

$$\varphi = a_1\pi \sin\frac{2\pi}{\tau}t\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right) + b_1\pi\cos\frac{2\pi}{\tau}t\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right) + a_2\Omega\sin\frac{2\pi}{\tau}t\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right) + b_2\Omega\cos\frac{2\pi}{\tau}t\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)$$

when a_1, b_1, a_2, b_2, are new constants, whose values are respectively,

$$\frac{a_1}{\underset{\varepsilon}{\sigma h}}, \ \frac{c_1 a_1}{\underset{\varepsilon}{\sigma h}}, \ \frac{c_2 a_1}{\underset{\varepsilon}{\sigma h}}, \ \frac{c_3 a_1}{\underset{\varepsilon}{\sigma h}},$$

or those multiplied by any arbitrary constant. This value of φ may be written in the form

$$\varphi = \left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)[(a_1\pi + a_2\Omega)\sin\frac{2\pi}{\tau}t + (b_1\pi + b_2\Omega)\cos\frac{2\pi}{\tau}t]$$

from which we obtain

$$u = \left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)\left\{\left(a_1\frac{d\pi}{dx} + a_2\frac{d\Omega}{dx}\right)\sin\frac{2\pi}{\tau}t + \left(b_1\frac{d\pi}{dx} + b_2\frac{d\Omega}{dx}\right)\cos\frac{2\pi}{\tau}t\right\}$$

$$v = \left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)\left\{\left(a_2\frac{d\pi}{dy} + a_2\frac{d\Omega}{dy}\right)\sin\frac{2\pi}{\tau}t + \left(b_1\frac{d\pi}{dy} + b_2\frac{d\Omega}{dy}\right)\cos\frac{2\pi}{\tau}t\right\}$$

$$w = -\sigma\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)[(a_1\pi + a_2\Omega)\sin\frac{2\pi}{\tau}t + (b_1\pi + b_2\Omega)\cos\frac{2\pi}{\tau}t]$$

We have, however,

$$\frac{d\pi}{dx} = \frac{d\pi}{d\theta}\cdot\frac{d\theta}{dx} = \frac{d\pi}{d\theta}\cdot\frac{d}{dx}\left(-\frac{\sigma^2r^2}{4}\right) = -\frac{\sigma^2 x}{2}\frac{d\pi}{d\theta}, \ \&c.\ldots\ldots$$

consequently,

$$u = -\frac{\sigma^2 x}{2}\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)\left\{\left(a_1\frac{d\pi}{d\theta} + a_2\frac{d\Omega}{d\theta}\right)\sin\frac{2\pi}{\tau}t + \left(b_1\frac{d\pi}{d\theta} + b_2\frac{d\Omega}{d\theta}\right)\cos\frac{2\pi}{\tau}t\right\}$$

$$v = -\frac{\sigma^2 y}{2}\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right)\left\{\left(a_1\frac{d\pi}{d\theta} + a_2\frac{d\Omega}{d\theta}\right)\sin\frac{2\pi}{\tau}t + b_1\frac{d\pi}{d\theta} + b_2\frac{d\Omega}{d\theta}\right)\cos\frac{2\pi}{\tau}t\right\}$$

From these equations we see that the path of the particle is always in a plane passing through itself and the axis of z.

The expression for p becomes now

$$p = p_0 + \rho g z_0 + 2\rho g\sigma\left\{\left(a_1\pi + a_2\Omega\right)\sin\frac{2\pi}{\tau}t + (b_1\pi + b_2\Omega)\cos\frac{2\pi}{\tau}t\right\}\frac{\overset{\sigma z_0}{\varepsilon} - \overset{-\sigma z_0}{\varepsilon}}{\underset{\varepsilon}{\sigma h} - \underset{\varepsilon}{\sigma h}}$$

The values which have been obtained for the displacements and the fluid pressure afford the complete solution of the problem under consideration.

The results obtained are, however, very much modified in the cases where the particles are removed to great distances from the axis. We have already seen the change produced in the function f in such a case, viz., this quantity becomes

$$f = \frac{a\cos s + b\sin s}{\sqrt{s}}$$

or since σ is a constant

$$f = \frac{\mathrm{A}\cos\sigma r + \mathrm{B}\sin\sigma r}{\sqrt{r}}.$$

We might have so transformed our first obtained value of f that the infinite series therein contained should have proceeded according to ascending powers of $\frac{1}{r^2}$ and thus obtained the same result; this, however, would have been a difficult process.

The quantities π, and Ω are now given by the equations,

$$\pi = \frac{\sin\sigma r}{\sqrt{r}}, \ \Omega = \frac{\cos\sigma r}{\sqrt{r}}$$

Substituting these in our value for φ and we have,

$$\varphi = \frac{1}{\sqrt{r}}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & + \ \varepsilon\end{matrix}\right\}$$

$$\left\{ (a_1 \sin \sigma r + a_2 \cos \sigma r) \sin \frac{2\pi}{\tau} t + (b_1 \sin \sigma r + b_2 \cos \sigma r) \cos \frac{2\pi}{\tau} t \right\}$$

for the simplest case of waves we can make $a_1 = b_2 = o$ and $a_2 = b_1 = a$; thus

$$\varphi = \frac{a}{\sqrt{r}} \left\{ \begin{matrix} \sigma(h-z) & -\sigma(h-z) \\ \varepsilon & + \varepsilon \end{matrix} \right\} \sin\left(\frac{2\pi}{\tau} t + \sigma r\right)$$

Differentiating this for r gives us the radial velocity of a particle, representing this by η and we have

$$\eta = \frac{1}{\sqrt{r}} \begin{pmatrix} \sigma(h-z) & -\sigma(h-z) \\ \varepsilon & + \varepsilon \end{pmatrix} \left\{ \sigma a \cos\left(\frac{2\pi}{\tau} t + \sigma r\right) + \frac{a}{2r} \sin\left(\frac{2\pi}{\tau} t + \sigma r\right) \right\}$$

discarding term containing $\frac{1}{r}$,

$$\eta = \frac{\sigma a}{\sqrt{r}} \begin{pmatrix} \sigma(h-z) & -\sigma(h-z) \\ \varepsilon & + \varepsilon \end{pmatrix} \cos\left(\frac{2\pi}{\tau} t + \sigma r\right)$$

also

$$w = -\frac{\sigma a}{\sqrt{r}} \begin{pmatrix} \sigma(h-z) & -\sigma(h-z) \\ \varepsilon & - \varepsilon \end{pmatrix} \sin\left(\frac{2\pi}{\tau} t + \sigma r\right)$$

The expression for p also becomes, after making the proper substitutions and reductions,

$$p = p_0 + \rho g z_0 + 2\rho g \frac{\sigma a}{\sqrt{r}} \frac{\begin{matrix} \sigma z_0 & -\sigma z_0 \\ \varepsilon & - \varepsilon \end{matrix}}{\begin{matrix} \sigma h & -\sigma h \\ \varepsilon & + \varepsilon \end{matrix}} \sin\left(\frac{2\pi}{\tau} t + \sigma r\right).$$

Introducing now the wave length l we have for great values of r

$$\frac{1}{\sqrt{r+l}} = \frac{1}{\sqrt{r}}$$

and also for a first approximation,

$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_0 + \eta}} = \frac{1}{\sqrt{r_0}}$$

we have, as before, $\sigma = \frac{2\pi}{l}$, and $l = \tau\, \omega$. Thus φ becomes now,

$$\varphi = \frac{a}{\sqrt{r_0}} \begin{pmatrix} \frac{2\pi}{l}(h-z) & -\frac{2\pi}{l}(h-z) \\ \varepsilon & + \varepsilon \end{pmatrix} \sin \frac{2\pi}{l}(wt + r)$$

with as before,

$$w = \sqrt{\frac{lg}{2\pi} \frac{\begin{matrix} \sigma h & -\sigma h \\ \varepsilon & - \varepsilon \end{matrix}}{\begin{matrix} \sigma h & -\sigma h \\ \varepsilon & + \varepsilon \end{matrix}}}$$

which for great depths or for particles near the surface becomes,

$$\sqrt{\frac{lg}{2\pi}}.$$

The same deductions are to be made here as in the case of plane waves, viz., that for particles any where within the mass of a fluid of infinite depth or near the surface of a mass of finite depth the velocity varies as the square root of the wave length. Write now $a\sigma = -a$ and collect all of our expressions:

$$\varphi = -\frac{l}{2\pi} \frac{a}{\sqrt{r_0}} \left\{ \begin{matrix} \frac{2\pi}{l}(h-z) & -\frac{2\pi}{l}(h-z) \\ \varepsilon & + \varepsilon \end{matrix} \right\} \sin \frac{2\pi}{l}(wt + r)$$

$$\eta = -\frac{a}{\sqrt{r_0}} \left\{ \begin{matrix} \frac{2\pi}{l}(h-z) & -\frac{2\pi}{l}(h-z) \\ \varepsilon & + \varepsilon \end{matrix} \right\} \cos \frac{2\pi}{l}(wt + r)$$

$$w = \frac{a}{\sqrt{r_0}} \left\{ \begin{matrix} \frac{2\pi}{l}(h-z) & -\sigma(h-z) \\ \varepsilon & - \varepsilon \end{matrix} \right\} \sin \frac{2\pi}{l}(wt + r)$$

$$p = p_0 + \rho g z_0 - 2\rho g \frac{a}{\sqrt{r_0}} \frac{\begin{matrix} \frac{2\pi}{l} z_0 & \frac{2\pi}{l} z_0 \\ \varepsilon & - \varepsilon \end{matrix}}{\begin{matrix} \frac{2\pi}{l} h & -\frac{2\pi}{l} h \\ \varepsilon & + \varepsilon \end{matrix}} \sin \frac{2\pi}{l}(wt + r)$$

$$w = \sqrt{\frac{lg}{2\pi} \cdot \frac{\begin{matrix} \frac{2\pi}{l} h & -\frac{2\pi}{l} h \\ \varepsilon & + \varepsilon \end{matrix}}{\begin{matrix} \frac{2\pi}{l} h & -\frac{2\pi}{l} h \\ \varepsilon & + \varepsilon \end{matrix}}}$$

We see from these expressions that the amplitudes of cylindrical waves differ only from those of plane waves by the factor $\frac{1}{\sqrt{r_0}}$—or in cylindrical waves the

amplitudes varies inversely as $\sqrt{r_0}$—and for particles very remote from the axis the amplitudes will vanish; whereas in the case of plane waves we saw that the amplitudes were always the same for the same depth.

From the expressions for the displacements we have as before

$$\frac{\eta^2}{\frac{a_2}{r_0}\left\{\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}+\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\right\}^2}+\frac{w^2}{\frac{a^2}{r_0}\left\{\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}+\underset{\varepsilon}{\frac{2\pi}{l}(h-z)}\right\}^2}=1$$

Now if h be finite we have as before for particles within the mass of the fluid—except near the surface—that they move in ellipses whose plane is vertical and passing through the wave axis, and whose transverse axis is in the direction of r. Also the axes of the ellipse decrease as r_0 increases and for particles infinitely remote from the wave axis they vanish, or these particles are at rest. The axes also, as in the case of plane waves, continuously decrease as z increases, and for $z-h$ the transverse axis becomes $\frac{2a}{\sqrt{r^0}}$ and the vertical axis vanishes as it should do.

If z be very small as compared with h, the equation of our ellipse becomes

$$\frac{\eta^2}{A^2-\underset{\varepsilon}{\frac{4\pi}{l}}z}+\frac{w^2}{A^2-\underset{\varepsilon}{\frac{4\pi}{l}}z}=1$$

when

$$A=\frac{a}{\sqrt{r_0}}\underset{\varepsilon}{\frac{2\pi}{l}}h$$

This is the equation of a circle whose radius is

$$A-\underset{\varepsilon}{\frac{2\pi}{l}}2$$

That is, for particles near the surface of a mass of fluid of finite depth, or for particles any where within the mass of a fluid of infinite depth, the motion is in a circle. It is shown as in the case of plane waves that this circular motion is uniform. In fact, all the results that we have obtained for plane waves are transferable into the corresponding results for cylindrical waves by merely multiplying by the factor $\frac{1}{\sqrt{r_0}}$.

Suppose now that we have a series of n waves of the same wave length and amplitude but of different phases, starting from the same axis; let these waves be defined in the same manner as in the case of plane waves and we shall have for the resultant wave function

$$\Sigma\varphi=-\frac{A}{\sigma\sqrt{r_0}}\left\{\underset{\varepsilon}{\sigma(h-z)}+\underset{\varepsilon}{\sigma(h-z)}\right\}\sin\sigma(\omega t+r+\phi)$$

when,

$$A^2=\sum_{i=0}^{i=n}a_i^2+2\sum_{j=0}^{j=n}a_j\sum_{=j+1}^{k=n}a_k\cos\sigma\left(\underset{k-1}{a}-\underset{k}{a}\right)$$

and

$$\Psi=\frac{1}{\sigma}\tan^{-1}\frac{\sum_{i=1}^{i=n}a_i\sin\sigma a_i}{\sum_{j=0}^{j=n}a_j\cos\sigma a_j}$$

If the wave lengths are the same, but the amplitudes different by reason of different initial values of r_0, the change in the form of these quantities is very slight; they become

$$A^2=\sum_{i=0}^{i=n}\frac{a_i^2}{\sqrt{r_i}}+2\sum_{j=0}^{j=n}\frac{a_j}{\sqrt{r_j}}\sum_{k=j+1}^{k=n}\frac{a_k}{\sqrt{r_k}}\cos\sigma\left(\underset{k-1}{a}-\underset{k}{a}\right)$$

and

$$\Psi=\frac{1}{\sigma}\tan^{-1}\frac{\sum_{i=1}^{i=n}\frac{a_i}{\sqrt{r_i}}\sin\sigma a_i}{\sum_{i=n}\frac{a_i}{\sqrt{r_j}}\cos\sigma a_j}$$

Suppose now that we have two waves of the same wave lengths and amplitudes, but of different phases and going in opposite directions. The resultant wave function will be

$$\varphi+\varphi'=-\frac{2a}{\sigma\sqrt{r_0}}\left\{\underset{\varepsilon}{\sigma(h-z)}-\underset{\varepsilon}{\sigma(h-z)}\right\}\sin\sigma\left(\omega t+\frac{a}{z}\right)\cos\sigma\left(r-\frac{a}{z}\right)$$

which corresponds to a standing wave. Differentiating for r and z we have for the displacement of η and w,

$$\eta=\frac{2a}{\sqrt{r_0}}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\varepsilon\end{matrix}\right\}\sin\sigma\left(\omega t+\frac{a}{z}\right)\sin\sigma\left(r-\frac{a}{z}\right)$$

$$w=\frac{2a}{\sqrt{r_0}}\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & -\varepsilon\end{matrix}\right)\sin\sigma\left(\omega t+\frac{a}{z}\right)\cos\sigma\left(r-\frac{a}{z}\right)$$

The ratio $\frac{\eta}{w}$ is independent of t, and we make the same deduction as before, that the particle move in right lines whose inclination to the axes varies with z and r.

Suppose that we have a series of parallel wave axes, and that waves proceed from them having the same length and the equal amplitudes but different phases. Let the wave functions be given as

$$\varphi_0=-\frac{a_0}{\sigma\sqrt{r_0^{(0)}}}\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\varepsilon\end{matrix}\right)\sin\sigma(\omega t+r^{(0)}+a_0)$$

$$\varphi_1=-\frac{a_1}{\sigma\sqrt{r_0^{(1)}}}\left(\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\varepsilon\end{matrix}\right)\sin\sigma(\omega t+r^{(1)}+a_1$$

$$\varphi_n=-\frac{a_n}{\sigma\sqrt{r_0^{(n)}}}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\varepsilon\end{matrix}\right\}\sin\sigma(\omega t+r^{(1)}+a_n)$$

adding these we have

$$\Sigma\varphi=-\frac{1}{\sigma}\sum_{i=o}^{i=n}a_i\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\varepsilon\end{matrix}\right\}\left\{\frac{\sin\sigma(\omega t+r^0+a_0)}{\sqrt{r_0^{(0)}}}+\frac{\sin\sigma(\omega t+r^{(1)}+a_1)}{\sqrt{r_0^{(1)}}}\right.$$
$$\left.\ldots.+\frac{\sin\sigma(\omega t+r^{(n)}+a_n)}{\sqrt{r_0^{(n)}}}\right\}$$

from which

$$w=\sum_{i=o}^{i=n}a_i\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & -\varepsilon\end{matrix}\right\}\left\{\frac{\sin\sigma(\omega t+r^{(0)}+a_1)}{\sqrt{r_0^{(0)}}}\right.$$
$$\left.\ldots.+\frac{\sin\sigma(\omega t+r^{(n)}+a_n)}{\sqrt{r_0^{(n)}}}\right\}$$

The waves may evidently so move that at certain points the vertical displacements shall be equal to zero. We can determine these points by placing the trigonometric factor of w equal to zero; thus

$$\frac{\sin\sigma(\omega t+r^{(0)}+a_0)}{\sqrt{r_0^{(0)}}}+\frac{\sin\sigma(\omega t+r^{(1)}+a_1)}{\sqrt{r_0^{(1)}}}$$
$$+\ldots.\frac{\sin\sigma(\omega t+r^{(n)}+a_n)}{\sqrt{r_0^{(n}}}=o$$

This expression can be divided into two parts, one of which shall have for a factor $\sin\sigma\,\omega\,t$, and the other $\cos\sigma\,\omega\,t$.

$$\sin\sigma\,\omega\,t\left\{\frac{\cos\sigma(r^{(0)}+a_0)}{\sqrt{r_0^{(0)}}}+\frac{\cos\sigma(r^{(1)}+a_1)}{\sqrt{r_0^{(1)}}}\right.$$
$$\left.+\ldots.\frac{\cos\sigma(r^{(n)}+a_n)}{\sqrt{r_0^{(n)}}}\right\}$$
$$+\cos\sigma\,\omega\,t\left\{\frac{\sin\sigma(r^{(o)}+a_0)}{\sqrt{r_0^{(o)}}}+\frac{\sin\sigma(r^{(1)}+a_1}{\sqrt{r_0^{(1)}}}\right.$$
$$\left.+\ldots.\frac{\sin\sigma(r^{(n)}+a_n)}{\sqrt{r^n{}_{(o)}}}\right\}=o$$

Equate separately to zero the factors multiplying $\sin\sigma\,\omega\,t$ and $\cos\sigma_0\,\omega\,t$, square and add the resulting equations and we have after some easy reductions,

$$\sum_{i=o}^{i=n}\frac{1}{r_i{}^{(i)}}+\sum_{j=o}^{j=n}\sum_{k=1}^{k=n}\cos\sigma\left\{\frac{(r^{(j)}-a_j)-(r_k-a_k)}{\sqrt{r_0{}^j\,r_0{}^k}}\right\}=o$$

For the simple case of $n=2$ or two wave axes we have since $a_0=o$

$$\frac{1}{r_0^{(o)}}+\frac{1}{r_0^{(1)}}+\frac{2}{\sqrt{r_0^{(o)}\,r_0^{(1)}}}\cos\sigma(r^{(o)}-r^{(1)}a_1)=o$$

If $r_0^{(o)}=r_0^{(1)}$ this becomes

$$\frac{2}{r_0}+\frac{2}{r_0}\cos\sigma\,(-a_1)=o$$

If now $-\sigma a_1=\mp(2n+1)\pi$ this equation will be satisfied, *i.e.*, if

$$a=\mp(2n+1)\,\frac{l}{2}$$

Therefore if the difference of phase is an odd multiple of half the wave length the vertical displacement is zero—but only for the points for which $r_0^{(0)}=r(_0)^{(1)}$. The points defined by the equation

$$r_0^{(0)}=r_0^{(1)}$$

lie on a plane which from its relation to the waves may be called the plane of symmetry. We will now examine a little more closely the conditions at this plane of symmetry. We have

$$\varphi_0=-\frac{a_0}{\sigma\sqrt{r_0}}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\varepsilon\end{matrix}\right\}\sin\sigma(wt+r)$$

$$\varphi' = -\frac{a_1}{\sigma\sqrt{r_0^{(1)}}}\left\{\begin{matrix}\sigma(h-z) & -\sigma(h-z)\\ \varepsilon & +\ \varepsilon\end{matrix}\right\}$$
$$\sin\sigma(\omega t + r' + a_1)$$

writing, for convenience, r' for $r^{(1)}$. We have also

$$u = \frac{d\varphi}{dr}\frac{dr}{dx} + \frac{d\varphi'}{dr}\frac{dr'}{dx}$$

$$v = \frac{d\varphi}{dr}\frac{dr}{dy} + \frac{d\varphi'}{dx'}\frac{dr'}{dy}$$

when $r^2 = x^2 + y^2$ and $r'^2 = (x-2a)^2 + y^2$. Now since

$$\sigma a_1 = \pm(2n+1)\pi$$

we will have for $r = r'$

$$\frac{d\varphi}{dr} = -\frac{d\varphi'}{dr'}$$

At the plane of symmetry $n = a$ so that

$$\frac{dr}{dx} = -\left(\frac{dr'}{dx}\right), \text{ and } \frac{dr}{dy} = \frac{dr'}{dy};$$

Therefore for $r = r'$ we have

$$u = 2\frac{d\varphi}{dx}\frac{dr}{dx},\ v = o,\ w = o.$$

That is, at the plane of symmetry the displacement perpendicular to it is twice as great as that due to either wave acting separately, and the displacements parallel to this plane and the vertical displacements are equal to zero. Suppose now, further, that $a_1 = o$: then we have for $r = r'$,

$$\frac{d\varphi}{dr} = \frac{d\varphi}{dr'};$$

therefore,

$$u = o,\ v = 2\frac{d\varphi}{dr}\frac{dr}{dy},\ w = 2\frac{d\varphi}{dz},$$

From which we have—if there is no difference of phase between the waves from the parallel axes—at the plane of symmetry there is no displacement in the direction of the axis of X i, e, in the direction perpendicular to this plane; also that the displacement parallel to the plane and the vertical displacement are twice as great as they would be if there was but one wave.

The reader who is interested in the subject of wave motion will do well to read an article on the subject by Lord Rayleigh in the April number of the *Philosophical Magazine* for 1876. An article in the September number of the same publication for 1878, though not bearing directly upon the subject, will also be found to contain much that is of value and interest; the article referred to is entitled "Hydrodynamic Problems in reference to the Theory of Ocean Currents," by M. Zöppritz. The mathematical theory of wave motion remains pretty much as Airy left it when he completed his work on the subject—so no better reference can be given than to that work—for any one wishing to acquire a thorough knowledge of the subject.

UNSINKABLE RIVER STEAMERS.

From "The Nautical Magazine."

Ever since iron has been in use as a material for ship-building, naval architects have been alive to the fact that although iron ships, as compared with wooden ones, are exceptionally liable to serious local damage, there is, on the other hand, a possibility of so constructing them, that such local damage shall not imperil the vessel. Much misapprehension has, however, obtained as to the arrangement, desirability, and uses of water-tight bulkheads in iron vessels. This was forcibly exemplified by the absurd legislation in respect of them, which formed a part of the Merchant Shipping Act of 1854, and which was wisely repealed in 1862. The 300th section of that Act prescribes that every steamer should be divided as nearly as possible into three parts by watertight bulkheads. Of course such a division was practically of no use whatever, as if one of the end compartments were pierced, the trim of the ship would be so much altered that, even if the water did not get to her hatchways, she could not live long in a seaway. As if still further to increase the absurdity of the whole thing, it was settled that the bulkheads need not extend to the upper deck in a vessel of more

than two decks. In nine out of ten cases of three-decked vessels, if one-third of the ship's displacement were lost, the water would certainly be higher than the top of the bulkheads and would then run into the other compartments and sink the vessel. Since that time the collision bulkhead, as it has been called, has been insisted on by the Registries, and has commended itself to shipowners generally, and many ships and their crews have been saved simply through having this provision in case of the vessel being pierced forward. A common fault is to place the collision bulkhead too far forward, but we believe that this is done less frequently now than it was a few years ago. In screw-steamers also, the idea of the water-tight compartment at the after-end has met with almost general approval, as affording a security in the frequent cases of injury to the propeller shaft which may be the cause of leakage.

A third application of the same principle is in the case of the double bottom, which has the additional advantage of affording the means for the use of water ballast. The water ballast tank has, in the past, mostly been confined to that part of the vessel forming the cargo hold, but now frequently extends the whole length of the ship.

The admiralty have constructed not only ironclads, but all war vessels in compartments, and in many cases the partitions are very numerous and the partitioned spaces very small. Most vessels of recent build have athwartship bulkheads, water tight flats, and also fore-and-aft bulkheads. War vessels are, of course, designed to encounter special risks and this extreme sub-division is specially useful to them.

There is, however, another class of vessels which have to encounter special risks, and in which we think the water-tight compartment principle might be much more usefully and easily applied than in sea-going merchant steamers. We refer to passenger steamers employed in the navigation of rivers and other smooth waters. These vessels are mostly built and fitted with a view to the conveyance of large numbers of deck passengers, the number being so large as to cause it to be altogether out of the question to attempt to provide for emergencies by life-saving apparatus. In many cases, it is true, the apparatus might be carried, but it would be useless in face of the fact that the crew of the steamer is so very small, sometimes almost infinitesimal compared with the passengers, and that the passengers, in vessels of this class, are usually especially helpless. Further, in many vessels which depend for a large part of their earnings on the conveyance of cargo, it has often been urged that water-tight partitions are a source of much inconvenience, and rather than put up with them the owners would give up the passenger trade. We think that perhaps the inconvenience has much been exaggerated, but it does certainly exist, and in reference to this we may mention a fact bearing upon the case which has recently come under our notice. The Admiralty have been enquiring into the capabilities of our large merchant steamers for being converted into useful crusiers in the event of war with a maritime power. Three hundred was the estimated number of vessels which, from their size and speed, were likely to be useful; and detailed inspection has shown that about one hundred of them fulfill the bulkhead condition, that is, are so far divided, that when in fighting trim, which, of course, is much less than their usual load draught, a shot hole in any one compartment would not be the cause of such extra immersion as to be a source of serious danger. The special point to which we refer, however, is, that many vessels otherwise suitable have failed, because of their owners objecting to having a bulkhead which cuts off the engine room from the boiler space. In the case of river passenger steamers the bulkheading of the hold need not be objected to from possible inconvenience, as regards either the stowage of cargo or easy communication between engines and boilers. These latter vessels do not usually lay themselves out for cargo, and what they do carry is in the form of miscellaneous goods which could as well, and as conveniently, go into small compartments, as into an undivided hold.

One serious drawback to compartments in seagoing vessels is the necessity for water-tight doors. There must for instance be such a door between the engine and boiler compartments, since to have them completely parted by a

bulkhead extending to the upper deck would be an intolerable nuisance, and even in supposable cases, a source of danger. When there are doors, the question is, will they act when wanted? Some of the vessels in the Royal Navy have been able to make good use of their water-tight sub-divisions, but in other cases the arrangement has altogether failed of its purpose. Our readers will remember the case of the *Agincourt*, which was run upon the Pearl Rock and got off again safely. The *Bellerophon* also sustained damage in a collision which would have been fatal to a vessel of ordinary construction. On the other hand the *Vanguard* and the German iron-clad *Grosser Kurfürst*, which was sunk by her consort in the English Channel, are instances of failure. There is first the liability of water-tight doors getting out of order so as not to act when wanted, and, secondly, the danger that they sometimes cannot be shut promptly after a collision has taken place. If, however, water-tight bulkheads were a part of the construction of smooth-water vessels, there would be no great need for doors at all, and it would certainly be safest to dispense with them. To go on deck to pass from one compartment to another, in a steamer which was never in a rough sea, would be always practicable, and in shallow vessels it could hardly be considered a very serious inconvenience.

It must further be remembered that in smooth-water steamers a much smaller number of sub-divisions would be sufficient than in the case of sea-going ships, and this for two reasons, viz.: that it is necessary for an ocean steamer to have a certain minimum free board, and that for her to be only sub-divided to such an extent as to be merely unsinkable, would be of little service to her, except she were very near a port. On the other hand, all that is necessary for a smooth-water steamer is that she should be insured against sinking: the smallest margin of buoyancy would give ample time in most cases to rescue all her passengers if not to save the vessel herself. Secondly, it it is well known that most river steamers, and more especially those which carry very large numbers of passengers, have a considerable margin of buoyancy, amounting in many instances to more than their total load displacement. For these reasons we should suppose that five bulkheads would usually be sufficient to ensure safety, even in the case of most extreme risk, that is, when the vessel is struck so near a bulkhead as to fill two adjoining compartments. This would necessitate a division between the engine and boiler rooms, and that the usual partitions at the ends of those spaces should be made securely water-tight. This only leaves a further necessity for the division of the fore and after portions of the vessel, each into two parts, certainly an inconvenience, perhaps a serious inconvenience, but nothing when compared with the resulting gain.

Before leaving this subject we must make some reference to the question of strength. It would, of course, be useless to construct a vessel unsinkable when two of her compartments are filled, and yet otherwise so weak that in such an emergency she would tear in two. In the event of any two adjoining compartments being pierced, a considerable longitudinal strain would be brought upon that part of the structure of the ship bounding and adjacent to the compartment into which the water was admitted. In most vessels floating, even in still water, the different parts of the hull are subjected to strains caused by the unequal distribution of weight and buoyancy. In a sailing ship having no cargo on board there is obviously an excess of buoyancy in the midship body, and the weight of the extreme ends is partly supported by the buoyancy amidships, thus bringing a strain upon the vessel, the tendency of which is to cause her to lose her sheer or to become *hogged*.

The extreme case of straining after a collision would be when the vessel was so struck that water was admitted to both engine and boiler rooms. Under these circumstances the weight of the engines and boilers, and of the structure of the ship near them, would be borne by the increased immersion of the ends, and it is obvious that very severe strains would be brought upon the midship body of the ship, the tendency of the strain being to alter the shape of the vessel, so that the buoyant ends should rise and the middle sink.

A NEW RULE FOR CALCULATING THE CONTENTS OF LAND SURVEYS.

By J. WOODBRIDGE DAVIS, C. E.

Written for Van Nostrand's Magazine.

Land surveying occupies but a small portion of the province of an engineer. When such work does present itself, he can meet it with numerous ready devices and rules, numerical and graphical, for determining areas circumscribed in the field.

The simplest of exact rules, hitherto presented to the world for this purpose, is the familiar method of *Double Meridian Distances*. Although this is otherwise named the *Pennsylvania Method*, it appears to have originated about a century ago in the mind of Thomas Burgh, of Ireland, of whom it is stated in preface to *Gibson's Surveying, New York*, 1834, that he received from the Irish Parliament twenty thousand pounds sterling for his discovery.

It is, however, on account of the infrequency of such work in the engineer's practice, that he often makes use of some instantly conceived rule for ascertaining areas, though it may be liable to mechanical errors, or may require much numerical labor, rather than commit to mind again the method for finding double meridian distances, although each several part of that excellent rule is as simple as arithmetic can be. With a desire to furnish a more easily remembered rule to the engineer, and at the same time to those who make land surveying a business, a method more vitally important in time-saving, the writer has endeavored, by applying to the case a principle he had already used in another branch of engineering, to substitute for the old a simpler and shorter rule, governing a process requiring, likewise, less labor, with what degree of success he leaves it to the judgment of those interested to decide.

To determine the area of a polygon surveyed, after finding the latitude and departure of each course as for ordinary method, proceed according to the following rule, which for convenience of reference shall be called C.

Rule C.

*Multiply the total latitude of each station by the sum of the departures of the two adjacent courses. The algebraic half sum of these products is the area.**

Take as an example that worked out on page 185 of *Gillespie's Land Surveying*.

The two methods are identical so far as the construction of the first seven columns of each. The work necessary to find double meridian distances for old method is not shown. No such outside work is required for the new method. To find the *total latitude* of each station, add to total latitude of preceding station the latitude of preceding course. If the latitude of last station, found in this way, be equal to the latitude of last course with reversed sign, the work is correct. However, the latitude of first station is

* This rule was used to the exclusion of the old method last Fall, at Columbia College, in both the School of Mines and the School of Arts.

Example of Old Method of Calculating Contents of Land Surveys.

Station	Bearing.	Distance.	Latitude.		Departure.		Double Mer. Distances.	Double Areas.	
			N+	S−	E+	W−		+	−
1	N 35° 00′ E	2.70	2.21	—	1.55	—	1.55	3.4255	—
2	N 83° 30′ E	1.29	.15	—	1.28	—	4.38	0.6570	—
3	S 57° 00′ E	2.22	—	1.21	1.86	—	7.52	—	9.0992
4	S 34° 15′ W	3.55	—	2.93	—	2.00	7.38	—	21.6234
5	N 56° 30′ W	3.23	1.78	—	—	2.69	2.69	4.7882	—
								2)21.8519	
								Square Chains	10.9259

EXAMPLE OF NEW METHOD OF CALCULATING CONTENTS OF LAND SURVEYS.

Station	Bearing.	Distance.	Latitude.		Departure.		Total Latitude	Adjacent Departures	Double Areas.
			N+	S−	E+	W−			
1	N 35° 00′ E	2.70	2.21	—	1.55	—	—	—	—
2	N 83° 30′ E	1.29	.15	—	1.28	—	2.21	2.83	6.2543
3	S 57° 00′ E	2 22	—	1.21	1.86	—	2.36	3.14	7.4104
4	S 34° 15′ W	3.55	—	2.93	—	2.00	1.15	−0.14	−0.1610
5	N 56° 30′ W	3.23	1.78	—	—	2.69	−1.78	−4.69	8.3482
									2)21.8519
									Square Chains 10.9259

always zero; of last station it is always the latitude of last course with reversed sign; and of second station it is always the latitude of first course. To find the *adjacent departures*, add the departures of the two courses, one on each side of station.

The advantages of this plan of calculation over the old method, are the following:

1°. *The rule governing the operation is brief, simple and easily remembered.*

2°. *The labor of computing double meridian distances is not needed. To find each total latitude or pair of adjacent departures, used instead of double meridian distances, only the sum of, or difference between, two numbers already in the table must be found. This work is done directly in table.*

3°. *Whereas by the old method a double area product must be found for every side of the polygon, by the new method the number of such products is always one less than the number of sides.* If one or more stations have same latitude as initial station, an equal number of other products disappear. If two alternate stations have same longitude, another product disappears.

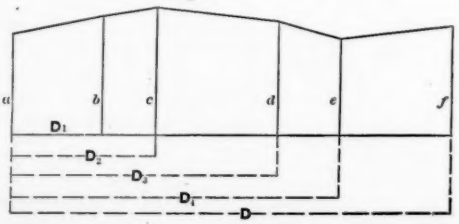
4°. *The factors, used in the method here described, for finding double area products, are almost always smaller than those used in the old method, sometimes containing fewer digits. The multiplications are, therefore, generally easier in this method than in the old.* The arithmetical aggregate of products of old method, for this example, is nearly 40. Of the present method this aggregate is 22. The average size of product by first rule is 7.9187. By second rule it is 5.5435.

Any station may be taken as the initial station, conveniently, the beginning point of survey. It is unnecessary to make a plot to determine most westerly or most easterly point, or for any other purpose connected with calculation of area. The most advantageous station, however, to which to refer the others, is that whose latitude is most nearly an average of the latitudes of all. This position insures the smallest double area factors.

The principle, upon which the foregoing rule may be made to depend, shall now be demonstrated, and a few of its applications shown. Although some of these have been already published at length by the writer, it seems proper to record them briefly, at least, in this connection, as the general rule appears to be new.

Consider any series of five trapezoids, lying consecutive with their bases on one straight line. The area of series, as found by ordinary method, the symbols of Fig. 1 used, is

Fig. 1



$$\tfrac{1}{2}[D_1(a+b)+(D_2-D_1)(b+c)+(D_3-D_2)(c+d)+(D_4-D_3)(d+e)+(D-D_4)(e+f')]. \quad (1)$$

This may be transformed into

$$\tfrac{1}{2}[D_1(a-c)+D_2(b-d)+D_3(c-e)+D_4(d-f')+D(e+f')]. \quad (2)$$

From this is derived the following rule, which is true, at least, for any series of five trapezoids arranged as in Fig. 1. For convenience let this be called RULE A.

RULE A.

To find the area bounded partly by any broken line, determined from a base line by means of rectangular ordinates, and otherwise bounded by the base line and terminal ordinates.

Multiply the distance of each intermediate ordinate from first end by the difference between the two adjacent ordinates, always subtracting the one more distant from the one less distant, as measured on broken line. Also, multiply distance of last ordinate from first by the sum of last two ordinates. Divide the sum of these products by 2.

If this rule apply to a series of n trapezoids, its area is

$$\tfrac{1}{2}[D_1(a-c)+D_2(b-d)+\text{etc.}+D_{n-1}(i-k)+D_n(j+k)]. \quad (3).$$

Add another trapezoid to the latter end of this series. Its area is

$$\tfrac{1}{2}(D_{n+1}-D_n)(k+l).$$

Add this to (3). The result is the area of a series of $n+1$ trapezoids, and is expressed by the following:

$$\tfrac{1}{2}[D_1(a-c)+\text{etc.}+D_{n-1}(i-k)+D_n(j-l)+D_{n+1}(k+l)].$$

Therefore, the rule applies to a series of $n+1$ trapezoids.

If from the series of n trapezoids the last trapezoid, whose area is

$$\tfrac{1}{2}(D_n-D_{n-1})(j+k),$$

be subtracted, the area of the remaining series of $n-1$ trapezoids is the following:

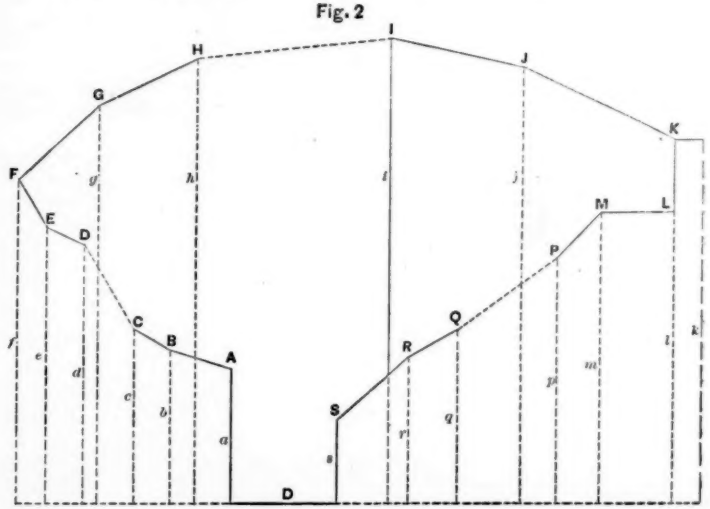
$$\tfrac{1}{2}[D_1(a-c)+D_2(b-d)+\text{etc.}+D_{n-1}(i+j)].$$

Therefore, the rule applies to a series of $n-1$ trapezoids.

It is certain that the rule applies to a series of five trapezoids. Let $n=5$. Then $n+1=6$, and $n-1=4$. Consequently, the rule applies to a series of six trapezoids, and to a series of four. For this reason it applies to series of three and seven trapezoids, and so on. Because a series containing any number of trapezoids may be found by continually adding, or subtracting, one trapezoid to, or from, a series of five, and because the application of the rule is not affected by each such operation, it follows that Rule A applies to a series consisting of any number of trapezoids.

To prove that the rule is perfectly general, whatever the *position* of trapezoids, let us consider a series in which

Fig. 2



portions of the broken line are retrogressive, or, in which some of the trapezoids are subtractive, as in Fig. 2, where the broken line is ABC . . . DEFGH . . . IJKLMP . . . QRS, and the terminal ordinates are a, s. The dotted lines between C and D, H and I, P and Q, represent omitted parts of broken line, which may be filled with any number of lines.

For convenience, let the distances from the ordinate a to the other ordinates, as b, j, be denoted by the same letters, respectively, with primes, as b', j'. Distances measured leftward from the initial ordinate, a, considered negative, the area between FGH . . . IJK, the ordinates, f, k, and the base line, is, by rule, whatever the number of trapezoids,

$$\tfrac{1}{2}[(g'-f')(f-h)+\text{etc.}+(j'-f')(i-k)+(k'-f')(j+k)].$$

This may be separated into two parts. One is

$$\tfrac{1}{2}[g'(f-h)+\text{etc.}+j'(i-k)+k'(j+k)]. \quad (4)$$

The other is, evidently, the product of $-\frac{1}{2}f''$ and the sum of all the ordinates from F to K, diminished by the product of $-\frac{1}{2}f'$ and the sum of all the same ordinates except the first two, f, g. This part is, therefore, equal to

$$-\tfrac{1}{2}f''(f+g). \quad (5)$$

The area included by FED . . . CBA and base line, between ordinates, f, a, is, by same rule, whatever the number of trapezoids,

$$\tfrac{1}{2}[(e'-f')(f-d)+\text{etc.}+(b'-f')(c-a)+(-f')(b+a)],$$

which, as before, is equivalent with

$$\tfrac{1}{2}[e'(f-d)+\text{etc.}+b'(c-a)-f'(f+e)]. \quad (6)$$

The area between SRQ . . . PML and base line, terminated by ordinates, s, l, is, similarly,

$$\tfrac{1}{2}[(r'-\mathrm{D})(s-q)+\text{etc.}+(m'-\mathrm{D})(p-l)+(l'-\mathrm{D})(m+l)],$$

or, $$\tfrac{1}{2}[r'(s-q)+\text{etc.}+m'(p-l)+l'(m+l)-\mathrm{D}(s+r)]. \quad (7)$$

To determine the entire area as described by rule, the sum of (6) and (7) must be subtracted from the sum of (4) and (5). This performed, the expression for area can be arranged thus:

$$\tfrac{1}{2}\left\{\begin{array}{c} b'(a-c)+\text{etc.}+e'(d-f')+f'(e-g) \\ +g'(f-h)+\text{etc.}+j'(i-k) \\ +k'(j-l)+l'(k-m)+m'(l-p) \\ +\text{etc.}+r'(q-s)+\mathrm{D}(r+s) \end{array}\right\} \quad (8)$$

From this it is seen that the rule applies to the series of trapezoids. This example explains the intention of the phrase, *as measured on broken line*, which occurs in the rule.

The broken line can be continued from S in any direction, through any kind of complication, forming the upper sides of new trapezoids. To all cases the same rule applies; because the portions of broken line can have no direction not considered in this example, and because, if the rule apply to a series of n trapezoids, it applies also to series of $n+1$ and of $n-1$ trapezoids.

There is one case, however, which on account of its frequency in practice, and because the formula for this case is reduced, deserves a special rule. This is the case where the end of the line, as S, coincides with the beginning, as A. Then, the broken line forms the perimeter of a polygon, and the result of the rule is, the area enclosed; while, because D becomes zero, the only term, $\mathrm{D}(r+s)$, in the formula for area, (8), which has for a factor the sum of two ordinates, vanishes.

To treat this case in the most general way, suppose B, C, D, E, F (Fig. 3) to be points anywhere situated in a plane and referred in position by rectangular co-ordinates to the point A in same plane; and suppose the points to be connected in order named by a broken line, ending again at B. Let the ordinates of the points be denoted by a, b, c, d, e, f, and the abscissæ by a', b', c', d', e', f'. Connect the origin by a straight line with any of the points, as B. Then the area of the polygon ABCDEFBA, which is equal to the area of the polygon BCD EFB, is by rule,

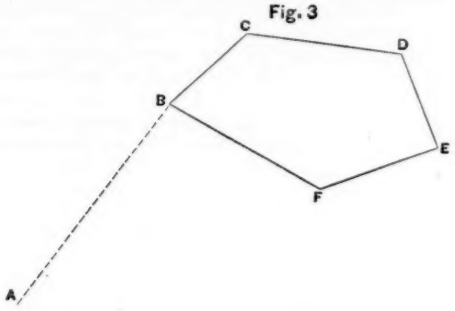
$$\tfrac{1}{2}[b'(a-c)+c'(b-d)+d'(c-e)+e'(d-f')+f'(e-b)+b'(f'-a)] \quad (9)$$

$$=\tfrac{1}{2}[b'(f-c)+c'(b-d)+d'(c-e)+e'(d-b)+f'(e-b)] \quad (10)$$

$$=-\tfrac{1}{2}[b(f'-c')+c(b'-d')+d(c'-e')+e(d'-f')+f(e'-b')] \quad (11)$$

From expressions (10), (11) can be framed the following rule:

Fig. 3



RULE B.

To find the area of any polygon whose vertices are fixed by rectangular co-ordinate measurement.

Multiply the abscissa of each vertex by the difference between the ordinates of the two adjacent vertices; or, multiply the ordinate of each vertex by the difference between the abscissæ of the two adjacent vertices; always making the subtraction in the same direction around the polygon. Half the sum of these products is the area.

For the sides CD, DE, EF of polygon can be substituted any number of sides. This causes expression (9) to contain more or fewer terms like its middle terms, but does not affect the end terms. Therefore, the reduction to forms of expressions (10), (11) can still be made. Consequently, the rule is true whatever the number of sides the polygon possesses.

Since the rule for a series of trapezoids is independent of shape of broken line, the rule for areas of polygons, which is identical with the former, is independent of the shape of perimeters.

Expressions (10), (11) show that the rule is independent of position of origin of co-ordinates.

The rule is independent of direction of co-ordinate axes, because the demonstration is independent of this.

It follows that the rule is perfectly general.

If the origin be placed at a vertex, one term vanishes whichever way the rule be used. If one of the co-ordinate axes be passed through two vertices, two terms vanish, when the rule is used one way.

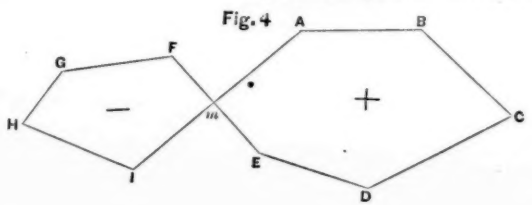
It is evident that Rule B is simply the case of Rule A when, the origin being at a vertex, the last and most difficult term vanishes; or, when the origin being not at a vertex, the last term vanishes, and of the remaining the two end terms reduce to one.

Sometimes the series of trapezoids, or the polygon, contains negative spaces. This always occurs when the boundary crosses itself. If, in Fig. 4, the broken line begin at A and follow the capital letters in alphabetical order, returning to A again, the area enclosed, as found by rule, is the area ABCDE *m* A, diminished by the area *m*IHGF*m*. But, if the broken line be considered to be ABCDE*m* IHGF*m*A, the area, by rule, is equal to the arithmetical sum of the areas of the two parts.

This can be explained as follows:

If a line enclose a space, the space is on one side only of the line; because, by definition, the line is a boundary. But, if the line cross itself, a portion of the line encloses new space on the other side of itself; that is, this portion not only does not enclose space of original description, but encloses space of directly opposite description. Hence, in this case, the

Fig. 4



amount of space of one description enclosed, is the amount enclosed on the proper side of the boundary less the amount enclosed on the other side of the boundary.

On the supposition that the line takes the course E*m*I, etc., the space is always on one side of the line, and is, consequently, all positive or all negative, accordingly as the line be supposed to follow the course around in the direction of the hands of a watch or in reverse direction. To accord with latter supposition, the polygon of Fig. 4 has eleven sides: to accord with first supposition, it is a polygon of nine sides.

The most general case of a series of trapezoids being considered, the series always contains negative spaces, when the broken line crosses itself, either end ordinate or the base line between end ordinates.

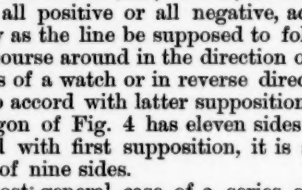
This case is met with in practice. A very practical example shall be given shortly. The following is an illustration from analytical geometry. The area between the axis of X and the line, $y=a-bx$, between the limits, $x=o$, $x=2a\div b$, is

$$\int_{x=o}^{x=2a\div b}(a-bx)dx=o.$$

But, arithmetically, the area is $a^2\div b$.

It is now seen that rule A, which always includes rule B, applies generally to all cases where areas are determined by dividing them into trapezoids and triangles, additive and subtractive, lying consecutive with their bases on one straight line. One general advantage of the rule is, therefore, that it reduces the solution of the most difficult cases to mere mechanical computation, saving us the labor of making plans and placing positive and negative trapezoids. Another advantage is that it makes the mechanical computation less; first, because it uses the differences between ordinates as factors, instead of sums of ordinates, and, second, because it often requires fewer terms than ordinary methods. These advantages appear more clearly, however, when illustrated by examples.

The case of such a series of trapezoids, as is shown in Fig. 1, is of frequent occurrence in all branches of engineering and in many other professions. Here rule A is invariably simpler than the ordinary method, except when the trapezoids are all of equal length. Let us apply the rule in the calculation of cubical contents of the borrow-pit, whose surface notes, the datum plane being fixed at 20 feet below elevation of station zero on base line, are the following:



The line on left is base line. The cross-sections are taken at right angles to this, 25 feet apart, as indicated. Each pair of numbers represent the elevation and distance from base line, of a point.

To estimate the mass of earth between this surface and datum plane, arrange distances in one column, as shown, and the differences of adjacent elevations in

TABLE OF OPERATIONS.

Sta.	Dist.	$h-h'$	Prods.	Pris. Cor.
0	10	10	50.	
	30	0	0	
	50	−15	− 375.	
	75	− 5	− 187 5	
	100	45	2250.	25
1	15	5	75.	0
	30	− 5	− 150.	25
	45	−10	− 450.	0
	70	− 5	− 350.	0
	100	40	4000.	0
2	15	− 2	− 30.	50
	40	−10	− 400.	− 30
For Sta. 1. {60×−8} {60×−5}	60	−13	− 780.	0
	100	45	4500.	0
3	20	− 5	− 100.	15
	35	− 8	− 280.	10
	75	− 5	− 375.	−120
	100	51	5100.	0
4	20	− 7	− 70.	0
	30	− 8	− 120.	0
	80	0	0	− 25
	100	57	2850.	0
			15157.5	− 50

Pris. Cor. ÷6= −8.33

4)15149.17

9)378729.25

6)42081.03

7013.5 cu. yds.

the next, except for each last distance, 100, opposite which place the sum of last two elevations. From these calculate the column of products, using half the products of first and last stations. Multiply the sum by the distance between stations, 25 feet, and divide by 54.

As this example is copied directly from a book on earthwork calculation—*Formulæ for R.R. Earthwork, Quantities and Average Haul*—it contains a little more matter than has been explained, namely, that exhibited in last column, and the equation, Pris. Cor. $\div 6 = -8.33$, which should be omitted here.

The advantage of this plan over that usually employed for this work is apparent at sight, and time shall not be spent here to analyze it. Especially to be noticed, however, is the fact that the factors of this method are more easily handled than those of other methods, since the ones in column of differences are nearly all composed of a single digit each (two in this example are zero), thus making unnecessary the extra work of separate multiplication, while the larger ones, opposite last distances, require for the operation, in present example, merely a shifting of decimal point. Where the ground is more level than this—the commoner case—this feature is more marked, and is independent of depth of pit.

Although it comes not within the scope of this article to explain the nature of the prismoidal correction, it may be mentioned, as an indirect advantage of this method, that the correction can be applied with great ease, furnishing a result exactly the same as if to each separate solid had been applied the prismoidal formula with the immense labor which has driven calculators, generally, from exact work. To do this, simply find difference between each distance belonging to each station, and the corresponding distance belonging to the next, and multiply this by the difference, taken inversely, between the two numbers opposite in column headed $h-h'$; divide sum of these products by six and add to column of *products*. Thus,

$(15-10)(10-5)=25$; $(30-30)[0-(-5)]=0$; etc.

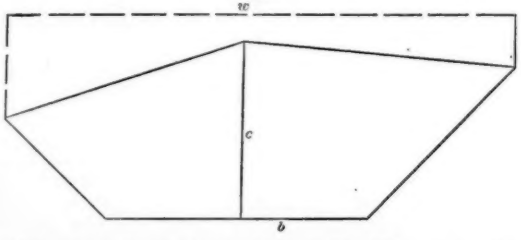
In this example, the new rule for summing trapezoids is applied in one direction only. If the cross sections were unequally distant from each other, the same rule could be advantageously applied in the direction of base line, too, as if the cross sections themselves were merely ordinates.

Because cross-sections of earthwork are generally calculated by means of trapezoids, the measurements in the field are commonly made from point to point, not from base line to each point, as shown in borrow-pit notes above. The latter method is, however, more convenient and less liable to errors. Often it is used by the tape-man to save time and trouble, even when he intends to use ordinary method of calculation. He stands on base line and pays out the tape to rodman, noting or announcing each distance, until the length of his tape or the change of level compels him to take a new position. With a hundred feet tape, in measuring above borrow-pit, he need never leave base line.

Rule B is useful in earthwork calculations. Let the following illustrate. It is well known, since the formula occurs in several late works, that the area of the ordinary "three-level" cross-section of railroad cut or fill, such as shown in Fig. 5, is

$$\tfrac{1}{2}w(c+sb)-sb^2, \qquad (12)$$

Fig. 5

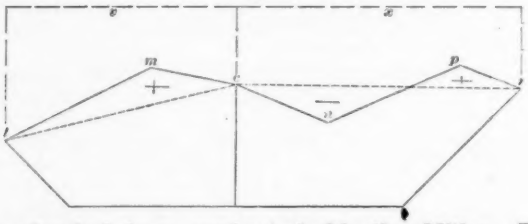


when w is the width of section between slope-stakes, c is the center cut or fill, b, the half-width of roadway, and s, the ratio of slope, equivalent to *vertical* $\div$

horizontal. But no algebraic formula, except the one to be derived here, has ever been given for the case of a cross-section, such as represented in Fig. 6, whose surface line has more than one change of direction. Merely the ordinary rule, directing to divide the area into trapezoids and triangles, additive and subtractive, has been employed. But a formula for all such cases can be very easily constructed by means of rule B.

Thus, let m, n, p represent the distances of the "breaks" from grade, m', n', p', the distances of same points from center line, r, l, the elevations of slope-stakes referred to grade, x, v, the side-widths, and c, b, s, as before. Draw lines, cr, cl. These represent surface line of a three level section, having, otherwise, same measurements as the irregular section. If, now, proceeding in the direction of the hands of a watch, with origin of measurements at grade in middle point of roadway, and considering the surface line of irregular section to be *above* the lines, lc, cr, we apply the rule to the polygon, $c\ n\ p\ r\ c\ l\ m\ c$, the result will be its area *above* the surface of three-level section, or the algebraic sum of all the positive and negative portions. If this be added to area of three-level section, the sum is the required area. Therefore, to expression (12) add

Fig. 6.



$$\tfrac{1}{2}[n'(c-p)+p'(n-r)+x(r-c) -v(c-m)-m'(l-c)]. \quad (13)$$

This sum is, since $w=v+x$,

$$\tfrac{1}{2}[v(m+sb)+x(p+sb)+m'(c-l)+ n'(c-p)+p'(n-r)]-sb^2. \quad (14).$$

From this comes the following

Rule to find the area of an irregular cross section:—

Multiply each side-width by the height of last break on that side added to $s\ b$. To the sum add the product of the distance of each break from center and the difference in elevation of the breaks next one on each side, always subtracting the one farther from center from the one nearer. Divide by 2, *and subtract* sb^2.

No terms in (13), whose coefficients are distances from center to breaks, are disturbed by the addition. Therefore, however many breaks there be, expression (14) maintains its shape. Hence the rule is perfectly general.

The advantages of this rule are seen in the following statements:

The number of terms in parenthesis in expression (14) is always equal to the number of trapezoids between surface line and grade, into which it is necessary to divide the area, by ordinary method of calculation; but the labor involved is less.

Instead of computing the areas of the two outside triangles and subtracting them, as by common method, the calculator need only deduct the quantity sb^2, which is the same for all the sections.

No diagrams are required to aid in the work. The field notes, kept according to this plan, are sufficient.

Rule A applies, more or less approximately, to a series of solids. The following illustrates its use.

Cross-sections on a railroad line are taken, as much as possible, at a constant distance apart, generally 100 feet. But at intervals these 100 feet solids require further subdivision. Let Fig. 1 represent a side view of one of these solids. D is its length, 100 feet, D_1, D_2, etc., are the distances of intermediate stations from first end, and a, b, c, etc., are the areas of cross-sections. Expression (2) now represents, approximately, the content of the whole 100 feet solid. This content is identical with that obtained by common rule, which directs to multiply sum of end areas by half distance between for content of each component solid.

The advantages of formula (2) are

here, as before, *first*, that differences between cross-sectional areas are much smaller factors than sums of these; *second*, that in calculating together the areas of each pair of cross-sections, by means of expressions (12), (14), the common term, $-sb^2$ vanishes, and one or more terms in each contain a common factor sb; and, *third*, that because one term of (2) has for a coefficient D, the full length of solid, this term may be used alone to represent the content of whole solid, which may then be calculated in one series with all the other 100 feet solids, and, afterwards, the remaining terms may be used as a correction.

The last is the chief advantage. Its value cannot, however, be appreciated without an example. It enables us to construct a single formula for the content of any possible entire bank or cutting, whatever the variety in lengths of component solids, and however irregular the cross-sections may be. This combines the advantages of rule A in both directions, and eliminates every constant term and factor. As in case of borrow-pits, the prismoidal correction can be easily applied.

Rule A has many other applications in earthwork calculations. In fact, it was devised for that class of computations; and this is the writer's excuse for exhibiting three of its examples in this article. Let us, however, now turn to a new field.

If for words, *abscissa*, *ordinate*, *vertex*, in rule B, be substituted *longitude*, *latitude*, *station*, that rule applies technically to the case of land surveys.

Frequently, perhaps, the rule in this shape will be serviceable in surveying. The writer was recently engaged on the survey of a large tract of coal land, containing about one-hundred-and-fifty square miles, which belonged mostly to one company, but was owned here and there by individual settlers. The latitudes and longitudes of all the corners were determined by lines run in all directions through the woods, and were tabulated. Thus, any parcel of land could be easily plotted, when wanted, in any of the geological or other maps; while merely a glance at the table served to indicate its position in the district When the area of such a piece was required, it was only necessary to proceed as in the following example:

Station.	Total latitude.	Total longitude.	Difference between alternate longitudes.	Double areas.
a	7087	94851	+1201	8,511,487
b	10020	97403	−6518	−65,310,360
c	8181	101369	−5752	−47,057,112
d	5012	103155	+2765	13,858,180
e	2873	98604	+8304	23,857,392

Area in sq. ft. = 66,140,413

If the latitudes were larger than longitudes, differences of latitudes should be made the factors with total longitudes.

When the determination of area is the only object, we may place the origin in most advantageous position, namely, at a station, thus getting rid of one term; and, instead of calculating the longitudes of all stations and finding differences between alternate ones, we may obtain the same result in simpler manner by adding the departures of each pair of adjacent courses. Modified in this way, the operation becomes governed by rule C, whose advantages have been already discussed.

Determinations of areas from diagrams, instead of numerical data, are frequently used. The common case is the polygon. Here rule B applies. Of course, to reduce work as much as possible, one axis should be passed through two vertices—conveniently, through one side. The importance of this case, warrants us in framing another rule by alteration of rule B.

Rule D.

To determine the area of a polygon in diagram:—

Perpendicular to one side prolonged, as base, draw ordinates from the vertices. Multiply each ordinate by the distance that the next following ordinate, as on perimeter, is to the right of next preceding ordinate. The algebraic half sum of these products is the area.

This requires one less distance to be measured, and contains one less term than the ordinary method of division into trapezoids. It is simpler also than the method of division into interior triangles, which is, perhaps, most commonly used. As an instance, consider the pentagon. By rule D, lay a straight edge on one side, and, sliding a triangle along this, draw three perpendiculars. By method of triangles, draw

two diagonals and three perpendiculars, all in different directions. Each method requires the same number of terms.

The writer has noticed several isolated instances wherein the formula for area of polygons has been demonstrated and used for special cases. The common rule for area of triangle is the case of formula (11) where the polygon has three vertices, and one axis passes through two. For the case of a triangle, one one of whose vertices is at origin, Julius Weisbach derived in section 112 of *Theoretical Mechanics*, an expression similar to formula (11). This enabled him to frame a very concise method for determining centers of gravity of polygons. Dr. Weisbach, in preface to first edition, makes particular mention of this section, together with a few others, as containing new matter peculiar to himself. In Gillespie's *Roads and Railroads*, p. 366, exists a formula for area of trapezium all of whose vertices are determined by measurements parallel with, and perpendicular to, one side. This formula contains two terms only, and is the case of formula (11), or more strictly of rule D, when one axis passes through two vertices. This formula was devised by E. M. Jenkins, C. E., now Registrar of Union College.

We are indebted to George Salmon for a demonstration of formulæ (10) (11) for all polygons. This occurs on some of the earlier pages of his *Conic Sections*.

The following is remarkable. Let us apply rule A, or its equivalent, formula (2), to a series of trapezoids of infinitesimal lengths—the differentials of any plane area. The trapezoids may vary in length to any extent. Thus, the abscissa may vary as any function of x, as u. Then the lengths of trapezoids are, according to position, the variable values of du. Let the corresponding ordinate vary as any other function of x, as y. Now, all the terms of formula (2), except the last, are represented by

$$\tfrac{1}{2}\int u[(y-dy)-(y+dy)]=-\int udy. \quad (15)$$

If we ascribe limits to this, the expression will represent the value of those terms for a definite series. The last term is, now,

$$\tfrac{1}{2}u[(y-dy)+y]=uy. \quad (16)$$

To this the same limits should be ascribed. The area is the sum of (15), (16).

But it is also $\int ydu$. Hence we obtain

$$\int ydu=uy-\int udy,$$

the well known *formula for integration by parts*, upon which so many general and special rules for integration depend.

A CONTRIBUTION TO THE HISTORY OF ELECTRIC LIGHTING.

By W. MATTIEU WILLIAMS, F.C.S., F.R.A.S.

From "The Journal of Science."

As the subject of lighting by electricity is occupying so much public attention, and the merits of various inventors and inventions are so keenly discussed, the following facts may have some historical interest in connection with it.

In October, 1845, I was consulted by some American gentlemen concerning the construction of a large voltaic battery for experimenting upon an invention, afterwards described and published in the specification of "King's Patent Electric Light" (Letters Patent granted for Scotland, November 26, 1845; enrolled March 25, 1846).

Mr. King was not the inventor, but he and Mr. Door supplied capital, and Mr. Snyder also held a share, which was afterwards transferred to myself. The inventor was Mr. Starr, a young man about 25 years of age, and one of the ablest experimental investigators with whom I have ever had the privilege of near acquaintance.

He had been working for some years on the subject, commencing with the ordinary arc between charcoal points. His first efforts were directed to maintaining constancy, and he showed me, in January of 1846, an arrangement by

which he succeeded in effecting an automatic renewal of contact by means of an electro-magnet, the armature of which received the electric flow, when the arc was broken, and which thus magnetized brought the carbons together and then allowed them to be withdrawn to their required separation, when the flow returned. This device was almost identical with that subsequently re-invented and patented by Mr. Staite (quite independently I believe), and which, with modifications, has since been rather extensively used.

Although successful so far, he was not satisfied. He reasoned on the subject, and concluded that the electric spark between metals, the electric arc between the carbons, and other luminous electric phenomena are secondary efforts due to the heating and illumination of electric carriers; that the electric spark of the conductors of ordinary electrical machines is simply a transfer of incandescent particles of metal, which effect a kind of electric convection, known as the disruptive discharge; and that the more brilliant arc between the carbon points is simply due to the use of a substance which breaks up more readily, and gives a longer, broader, and more continuous stream of incandescent convection particles.

This is now readily accepted, but at that time was only dawning upon the understanding of electricians. I am satisfied that Mr. Starr worked out the principle quite originally. He therefore concluded that, the light being due to solid particles heated by electric disturbance, it would be more advantageous—as regards steadiness, economy, and simplicity—to place in the current a continuous solid barrier, which should present sufficient resistance to its passage to become bodily incandescent without disruption.

This was the essence of the invention specified in King's Patent as "a communication from abroad," which claims the use of continuous metallic and carbon conductors, intensely heated by the passage of a current of electricity, for the purpose of illumination.

The metal selected was platinum, which, as the specification states, "though not so fusible as iridium, has but little affinity for oxygen, and offers a great resistance to the passage of the current." The form of thin sheets known by the name of leaf-platinum is described as preferable. These to be rolled between sheets of copper in order to secure uniformity, and to be carefully cut in strips of equal width, and with a clean edge, in order that one part may not be fused before the other parts have obtained a sufficiently high temperature to produce a brilliant light. This strip to be suspended between forceps.

I need not describe the arrangement for regulating the distance between the forceps, for directing the current, &c., as we soon learned that this part of the invention was of no practical value, on account of the narrow margin between efficient incandescence and the fusion of the platinum. The experiments with the large battery that I made—consisting of 100 Daniell cells, with 2 square feet of working surface of each element in each cell, and the copper-plates about $\frac{3}{4}$ of an inch distant from the zinc—satisfied all concerned that neither platinum nor any available alloy of platinum and iridium could be relied upon; especially when the grand idea of subdividing the light by interposing several platinum strips in the same circuit, and working with a proportionally high power, was carried out.

This drove Mr. Starr to rely upon the second part of the specification, viz., that of using a small stick of carbon made incandescent in a Torricellian vacuum. He commenced with plumbago, and, after trying many other forms of carbon, found that which lines gas-retorts that have been long in use was the best.

The carbon stick of square section, about one-tenth of an inch thick and half an inch working length, was held vertically, by metallic forceps at each end, in a barometer tube, the upper part of which, containing the carbon, was enlarged to a sort of oblong bulb. A thick platinum wire from the upper forceps was sealed into the top of the tube and projected beyond; a similar wire passed downwards from the lower forceps, and dipped into the mercury of the tube, which was so long that when arranged as a barometer the enlarged end containing the carbon was vacuous.

Considerable difficulty was at first encountered in supporting this fragile

stick. Metallic supports were not available, on account of their expansion; and, finally, little cylinders of porcelain were used, one on each side of the carbon stick, and about three-eighths of an inch distant.

By connecting the mercury cup with one terminal of the battery, and the upper platinum-wire with the other, a brilliant and perfectly steady light was produced, not so intense as the ordinary disruption arc between carbons, but equally if not more effective, on account of the magnitude of brilliant radiating surface.

Some curious phenomena accompanied this illumination of the carbon. The mercury column fell to about half its barometric height, and presently the glass opposite the carbon stick became slightly dimmed by the deposition of a thin film of sooty deposit.

At first, the depression of the mercury was attributed to the formation of mercurial vapor, and is described accordingly in the specification; but further observation refuted this theory, for no return of the mercury took place when the tube was cooled. The depression was permanent. The formation of vaporous carbon was suggested by one of the capitalists; but neither Mr. Starr nor myself was satisfied with this, nor with any other surmise we were able to make during Mr. Starr's lifetime, nor up to the period of final abandonment of the enterprise.

When this occurred, the remaining apparatus was assigned to me, and I retained possession of the finally arranged tube and carbon for many years, and have shown it in action worked by a small Grove's battery in the Town Hall of Birmingham, and many times to my pupils at the Birmingham and Midland Institute.

These exhibitions suggested an explanation of the mysterious gaseous matter, which I believe to be the correct one, and also of the carbon deposit. It is this:—That the carbon contains occluded oxygen; that when the carbon is heated, some of this oxygen combines with the carbon, forming carbonic oxide and carbonic acid, and a little smoke. I proved the presence of carbonic acid by the usual tests, but did not quantitatively determine its proportion of the total atmosphere.

If I were fitting up another tube on this principle, I should wash it with a strong solution of caustic potash before filling with mercury, and allow some of the potash solution to float on the mercury surface, by filling the tube while the glass remained moistened with the solution. My object would be to get rid of the carbonic acid as soon as formed, as the observations I have made, lead me to believe that—when the carbonic stick is incandescent in an atmosphere of carbonic acid or carbonic oxide—a certain degree of dissociation and re-combination is continually occurring, which weakens and would ultimately break up the carbon stick, and increases the sooty deposit.

The large battery above described was arranged for intensity, but even then it was found that the quantity (I use the old-fashioned terms) of electricity was excessive, and that it worked more advantageously when the cells were but partially filled with acid and sulphate. A larger stick of carbon might have been used with the whole surface in full action.

After working the battery in various ways, and duly considering the merits of the other forms of battery then in use, Mr. Starr was driven to the conclusion that for the purposes of practical illumination the voltaic battery was a hopeless source of power, and that magneto-electric machinery driven by steam-power must be used. I fully concurred with him in this conclusion, so did Mr. King, Mr. Dorr, and all concerned.

Mr. Starr then set to work to devise a suitable dynamo-electric machine, and, following his usual course of starting from first principles, concluded that all the armatures hitherto constructed were defective in one fundamental element of their arrangement. The thick copper-wire surrounding the soft iron core necessarily follows a spiral course, like that of a coarse screw thread; but the electric current or lines of force which it is designed to pick up and carry circulate at right angles to the axis of the core, and extend to some distance beyond its surface. The problem thus presented is to wind around the soft iron a conductor that shall be broad enough to grasp a

large proportion of this outspread force, and yet shall follow its course as nearly as possible by standing out at right angles to the axis of the armature. This he proposed to effect by using a core of square section, and winding round it a broad ribbon of sheet copper, insulated on both sides by cementing on its surfaces a layer of silk ribbon. This armature to be laid with one edge against one side of the core, and carried on thus to the angle; then turned over so that its opposite edge should be presented to the next side of the core; this side to be followed in like manner, the ribbon similarly turned again at the next corner, and so on till the core becomes fully enclosed or armed with the continuous ribbon, which would thus encircle the core with its edges outwards, and nearly at right angles to the axis, in spite of its width, which might be increased to any extent found by experiment to be desirable.

At this stage my direct co-operation and confidential communication with Mr. Starr ceased, as I remained in London while he went to Birmingham, in order to get his machinery constructed, and to apply it at the works of Messrs. Elkington, who had then recently introduced the principle of dynamo-electric motive-power, electro-plating, &c., and were, I believe, using Woolrich's apparatus, the patent for which was dated August 1, 1842, and enrolled February 1, 1843.

I am unable to state the results of his efforts in Birmingham. I only heard the murmurs of the capitalists, who loudly complained of expenditure without results. They had dreamed the same dream that Mr. Edison has recently re-dreamed, and has told the world so loudly. They supposed, that the mechanically-excited current might be carried along great lengths of wire, and the carbons interposed where required, and that the same electricity would flow on and do the duty of illumination over and over again, as a river may fall over a succession of weirs and turn water-wheels at each. Mr. Starr knew better; his scepticism was misinterpreted; he was taunted with failure and non-fulfillment of the anticipations he had raised, and with the fruitless expenditure of large sums of other people's money. He was high-minded, honorable, and a very sensitive man, suffering already from overworked brain before he went to Birmingham. There he worked again still harder, with further vexation and disappointment, until one morning he was found dead in his bed. Having, during my short acquaintance with him, enjoyed his full confidence in reference to all his investigations, both completed and incomplete, I have no hesitation in affirming that his early death cut short the career of one, who, otherwise, would have largely contributed to the progress of experimental science, and have done honor to his country. His martyrdom, for such it was, taught me an useful lesson I then much needed, viz., to abstain from entering upon a costly series of physical investigations without being well assured of the means of completing them, and, above all, of being able to afford to fail.

There are many others who sorely need to be impressed with the same lesson, especially at this moment and in connection with this subject.

The warning is the most applicable to those who are now misled by a plausible but false analogy. They look at the progress made in other things, the mighty achievements of modern Science, and therefore infer that the electric light —even though unsuccessful hitherto— may be improved up to practical success, as other things have been. A great falacy is hidden here. As a matter of fact, the progress made in electric lighting since Mr. Starr's death, thirty-one years ago, has been very small indeed. As regards the lamp itself, no progress whatever has been made. I am satisfied that Starr's continuous carbon stick, properly managed in a true vacuum, or an atmosphere free from oxygen, carbonic oxide, carbonic acid, or other oxygen compound, is the best that has yet been placed before the public for all purposes where exceptionally intense illumination (as in lighthouses) is not demanded. It is the steadiest, the cheapest, and least glaring in proportion to the amount of light it radiates. It has not been "pushed" like other devices, simply because it is nobody's exclusive property.

Comparing electric with gas lighting, the hopeful believer in progressive improvement appears to forget that gas making and gas lighting are as susceptible of further improvement as electric

lighting, and that, as a matter of fact, its practical progress during the last forty years is incomparably greater than that of the electric light. I refer more particularly to the practical and crucial question of economy. The by-products, the ammoniacal salts, the liquid hydrocarbons, and their derivatives, have been developed into so many useful forms by the achievements of modern chemistry, that these, with the coke, are of sufficient value to cover the whole cost of manufacture, and leave the gas itself as a volatile residuum that costs nothing. It would actually and practically cost nothing, and might be profitably delivered to the burners of gas consumers (of far better quality than now supplied in London) at one shilling per thousand cubic feet, if gas making were conducted on sound commercial principles,—that is, if it were not a corporate monopoly, and were subject to the wholesome stimulating influence of free competition and private enterprise. As it is, our gas and the price we pay for it are absurdities, and all calculations respecting the comparative cost of new methods of illumination should be based not on what we *do* pay per candle-power of gas-light, but what we *ought* to pay and *should* pay if the gas companies were subjected to desirable competition, or visited with the national confiscation I consider they deserve.

Having had considerable practical experience in the commercial distillation of coal for the sake of its liquid and solid hydrocarbons, I speak thus plainly and with full confidence.

There is yet another consideration, and one of vital importance, to be taken into account, viz., that—whether we use the electric light derived from a dynamo-electric source, or coal-gas—our primary source of illuminating power is coal, or rather the chemical energy derivable from the combination of its hydrogen and carbon with oxygen. Now this chemical energy is a limited quantity, and the progress of Science can no more increase this quantity than it can make a ton of coal weigh 21 cwts. by increasing the quantity of its gravitating energy.

The demonstrable limit of scientific possibilities is the economical application of this limited store of energy, by converting it into the demanded form of force without waste. The more indirect and roundabout the method of application, the greater must be the loss of power in the course of its transfer and conversion. In heating the boiler that sets the dynamo-electric machine to work, about one-half the energy of the coal is wasted, even with the best constructed furnaces. This merely as regards the quantity of water evaporated.

In converting the heat force into mechanical power—raising the piston, &c., of the steam-engine—this working half is again seriously reduced. In further converting this residuum of mechanical power into electrical energy, a further and considerable loss is suffered in originating and sustaining the motion of the dynamo-electric machine in the dissipation of the electric energy that the armature cannot pick up, and in overcoming the electrical resistances to its transfer.

I am unable to state the amount of this loss in trustworthy figures, but should be very much surprised to learn that, with the best arrangements now known, more than one-tenth of the original energy of the coal is made practically available. This small illuminating residuum may, and doubtless will, be increased by the progress of practical improvement; but, from the necessary nature of the problem, the power available for illumination at the end of the series must always be but a small portion of that employed at the beginning.

In burning the gas derived from coal, we obtain its illuminating power *directly*, and if we burn it properly we obtain nearly all. The coke residuum is also directly used as a source of heat. The chief waste of the original energy in the gas-works is represented by that portion of the coke that is burned under the retorts, and in obtaining the relatively small amount of steam-power demanded in the works. These are far more than paid for by the value of the liquid hydrocarbons and the ammonia salts, when they are properly utilized.

In concluding my narrative I may add, that, after Mr. Starr's death, the patentees offered to engage me on certain terms to carry on his work. I declined this, simply because I had seen enough

to convince me of the impossibility of any success at all corresponding to their anticipations. During the intervening thirty years, I have abstained from further meddling with the electric light, because all that I had seen then, and have heard of since, has convinced me that—although as a scientific achievement the electric light is a splendid success—its practical application to all purposes where cost is a matter of serious consideration is a complete and hopeless failure, and must of necessity continue to be so.

Whoever can afford to pay some shillings per hour for a single splendid light of solar completeness can have it without difficulty, but not so where the cost in pence per hour per burner have to be counted.

I should add that before the publication of King's specification, Mr. (now Sir William) Grove proposed the use of a helix or coil of platinum, made incandescent by electricity, as a light to be used for certain purposes. This was shown at the Royal Society on or about December 1st, 1845.

AN ULTRA-GASEOUS STATE OF MATTER.

From "The English Mechanic."

PROBABLY the most remarkable contribution to modern science that the present year has witnessed is the paper "On the Illumination of Lines of Molecular Pressure, and the Trajectory of Molecules," by Mr. W. Crookes, F.R.S., read at the meeting of the Royal Society, on Dec. 5. It is not unknown to our readers that Mr. Crookes has been engaged for many years in the study of molecular physics, the radiometer being one of the results of that study, and, as it has turned out, a means to more important discoveries. Last year Pictet and Cailletet solidified what were known as the permanent gases, and now Mr. Crookes has demonstrated the fact that, under certain conditions, gases may become so far changed, both in physical constitution and properties, as to form a fourth state of matter. At least the distinguished assembly before whom he performed his experiments can have no doubt that, just as below the gaseous state there is the liquid and the solid, so above the liquid there is the gaseous and the ultra-gaseous, or ethereal. Mr. Crookes has not arrived at this important discovery *per saltum*, and he was not surprised when he found means of demonstrating the truth of an hypothesis he has more than once shadowed out in papers on the radiometer. In his latest paper he commences by referring to his examination of the dark space which appears round the negative pole of an ordinary vacuum tube when a spark from an induction coil is passed through. This space has been examined many times, under differing conditions; and Mr. Crookes has been able to arrive at several propositions. Thus, the setting up, by electrical means, of an intense molecular vibration in a disc of metal excites a molecular disturbance, which affects the surface of the disc and the surrounding gas. With a dense gas the disturbance extends a short distance only from the metal; but as rarefaction continues, the layer of molecular disturbance increases in thickness. In air, at a pressure of .078 mm., it extends for at least 18 mm. from the surface of the disc, and forms an oblate spheroid around it. The diameter of this dark space varies with the degree of exhaustion, the kind of gas, the temperature of the negative pole, and, to a slight extent, with the intensity of the spark. It is greatest in hydrogen, and least in carbonic acid, as compared with air under equal degrees of exhaustion. The shape and size of this dark space do not vary with the distance separating the poles, nor—except slightly—with alteration of battery power, or intensity of spark. When the power is great, the brilliancy of the other parts of the tube overpowers the dark space, rendering it difficult of observation. To determine whether this visible layer of molecular disturbance is identical with the invisible layer of molecular pressure or stress, Mr. Crookes has made many ex-

periments, one of which is accomplished by the aid of the electrical radiometer. An ordinary radiometer, with aluminium discs (coated on one side with a film of mica) for vanes is constructed in such a manner that an electrical current can be passed through the fly. Instead of a glass cup the fly is supported on a hard steel cup, and the needle-point on which it works is connected, by means of a wire, with a platinum terminal sealed into the glass. At the top of the bulb a second terminal is sealed in, and the radiometer can therefore be connected to an induction coil in such a manner that the movable fly becomes the negative pole. When connected with the coil a halo of velvety violet light is seen on the metallic side of the vanes, the mica side remaining dark. As the pressure increases a dark space begins to separate the violet halo from the metal, and at a pressure of .5mm. it extends to the glass, and positive rotation commences. On increasing the exhaustion the dark space widens out, and appears to flatten itself against the glass, the rotation becoming very rapid. Somewhat similar effects are witnessed when aluminium cups are used instead of discs for the vanes, the dark space retaining the shape of the cup almost exactly. The bright margin of the dark space becomes concentrated at the concave side of the cup to a luminous focus, and widens out at the convex side, touching the glass as the exhaustion is increased, when rotation commences, and becomes very rapid as the dark space further increases in size. This convergence of the lines of force to a focus attracted special attention, and another instrument, having the cup-shaped negative pole fixed, was devised for the examination of the phenomena, one of which was of an entirely novel and highly interesting nature. At very high exhaustions (those known as "Crookes' vacua," measured by a few millionths of an atmosphere) the dark space becomes so large that it fills the tube. The dark violet focus is still visible to the careful observer; but the rays diverging from this focus produce upon the part of the glass where they fall a spot of greenish-yellow light. If now a more perfect vacuum is obtained, the whole bulb becomes illuminated with a beautiful phosphorescent light, which is greenish-yellow because of the peculiar composition of the soft German glass used in the experiments. Other qualities of glass give other colors, the phosphorescence being due to the fact that the gaseous molecules are practically unimpeded in their movements, and impinge upon the glass with sufficient velocity to produce light, which is distinguished from the ordinary light of vacuum tubes by the following characteristics: The green focus cannot be seen in the space of the tube, but only where the projected beam strikes the glass. The position of the positive pole makes scarcely any difference to the direction and intensity of the lines of force producing the green light. The spectrum is a continuous one, most of the red and the higher blue rays being absent, while the spectrum of the light observed in the tube at lower exhaustions is characteristic of the residual gas. No difference can be detected in the green light by spectrum examination, whether the residual gas be nitrogen, hydrogen, or carbonic acid. The green phosphorescence commences, besides, at a different degree of exhaustion in different gases. The viscosity of a gas is almost as persistent a characteristic of its individuality as its spectrum; but Mr. Crookes finds that when the spectral and other characteristics of the gas begin to disappear, the viscosity also commences to decline, and at an exhaustion at which the green phosphorescence is most brilliant the viscosity has sunk to an insignificant amount. The rays exciting green phosphorescence will not turn a corner in the slightest degree, but radiate from the negative pole in straight lines, casting strong and sharply-defined shadows from objects which happen to be in their path. On the contrary, the ordinary luminescene of vacuum tubes will travel hither and thither along any number of curves and angles. By means of beautifully constructed specimens of ingenious apparatus Mr. Crookes exhibited the projection of molecular shadows, and demonstrated that they were molecular, not optical, although rendered apparent by optical appliances. From a consideration of the results of his experiments he advances the theory that the induction spark illuminates the lines of molecular pressure caused by the electrical excitement of the negative pole, and he gives

the following explanation: "The thickness of the dark space is the measure of the mean length of the path between successive collisions of the molecules. The extra velocity with which the molecules rebound from the excited negative pole keeps back the more slowly moving molecules which are advancing towards the pole. The fight occurs at the boundary of the dark space where the luminous margin bears witness to the energy of the collisions of the molecules. When the exhaustion is sufficiently high for the mean length of path between successive collisions to be greater than the distance between the electrode and the glass, the swiftly moving rebounding molecules spend their force, in part or in whole, on the sides of the vessel, and the production of light is the consequence of this sudden arrest of velocity."

We have not space to give an account, even in outline, of all the experiments and all the phenomena which Mr. Crookes describes in his paper, nor is it necessary, as it is sure to find its way into the hands of all students of physics. The theory propounded at the conclusion is, however, as we have said, one of the most remarkable discoveries of modern times, involving as it does the demonstration of a fourth state of matter. The modern idea of the gaseous state—the kinetic theory of gases—is that a given space contains millions of millions of molecules in rapid motion in all directions, each having millions of encounters in a second. In such a supposition the length of the mean free path of the molecules is exceedingly small as compared with the dimensions of the vessel, and the conditions are such that the constant collisions, which are the essential element of the gaseous state, can occur. But, under high degrees of exhaustion the free path is made so long that the number of collisions in a given time is small compared to the number of misses, and the average molecule being allowed to obey its own motions without interference, the properties constituting gaseity are reduced to a minimum, and the matter becomes exalted to an ultra-gaseous state, in which phenomena are seen, hitherto unknown and at present not comprehended. The phenomena discovered by Mr. Crookes in his exhausted tubes reveal to physical science a new world—a world where, for instance, the corpuscular theory of light holds good, and where light does not always move in straight lines, but where "we can never enter, and in which we must be content to observe and experiment from the outside." Perhaps the most astonishing of the many experiments performed by Mr. Crookes is that in which he showed that great heat is evolved when the concentrated focus of rays from an aluminium cup is deflected sideways to the walls of the glass tube by a magnet. The heat is sufficient to fuse platinum, but in the experiment shown at the Royal Society, to avoid destroying the apparatus, a luminous spot of incandescence was made to travel over the platinum foil by applying the magnet in different positions. This experiment was suggested by the discovery, while examining the green phosphorescent light produced by the focus of concentrated rays, that the glass tube at that spot became so hot as to burn the finger. Thus again at the end of the year we have to announce a great advance in physical science.

The Belgian State Railroads, though increasing in mileage and in equipment, make smaller and smaller profits per train mile and per mile of road. The proportion of expenses to receipts is much greater than formerly. Some time ago several railroads were leased for fifty per cent. of their gross earnings, thinking that it was a fair bargain. Since then the expenses of its whole system, of which these leased roads are generally the less important parts, have one year been up to sixty-eighth per cent., and they seem unlikely again to be as low as sixty per cent. The Government roads seem to be extraordinarily well stocked, for there are 1066 locomotives and 33,395 cars of all kinds for the 1339 miles of road. It is hard to believe that there are not more than are necessary; for while the locomotives made an average mileage of 24,009 miles in 1872, they made but 18,588 in 1877, the number having increased in that time from 638 to 1066.

—*The Engineer.*

THE STABILITY OF DOCK WALLS.

By ROMILLY ALLEN, A. I. C. E.

Contributed to VAN NOSTRAND'S MAGAZINE.

In a paper read before the Edinburgh and Leith Engineers' Society on the 5th of February, Mr. Romilly Allen, A.I.C.E., gave the following particulars of a new method of calculating the stresses on a Dock Wall:

The forces tending to displace a Dock Wall are:

Firstly. A Pressure of Water, acting normally to the face, and tending to push it backwards.

Secondly. A Thrust of Earth, acting in a direction parallel to the surface of the ground, tending to push it forwards.

Thirdly. The weight of the Masonry and of the Earth supported on the stepping behind, tending to pull it vertically downwards.

Since the wall is required to stand when the dock is empty, the pressure of the water will be neglected as being an element of safety when the dock is full.

The portion of the wall dealt with in the following investigation is a cross section of one foot in length.

The weight of the wall acts through its center of gravity, which is found thus:

Cut out the cross section in thick drawing paper of uniform texture. Mark a vertical line on a drawing-board placed upright, by means of a plumb bob. Hang up the section against the board by a pricking point stuck through one corner, and let it swing. When it has come to rest, mark where the vertical line cuts. Repeat the process and join the points so found with the respective points of suspension. The intersection of the lines will be the center of gravity required. With a section drawn to a scale of $\frac{1}{4}$ inch to the foot (which is a convenient size) the center of gravity, without the error of deviation from the true position exceeding one inch in magnitude. If the weight of the earth, which rests on the stepping behind the wall, is taken into account, its center of gravity must also be found in the same way and the center of gravity of the compound mass of earth and masonry be deduced therefrom.

The most convenient unit of weight to take is that of 1 cubic foot of masonry, as the number of square feet in the cross section will then represent the weight of the wall per foot forward. The earth thrust has now to be considered.

Its center of pressure is at a point $\frac{1}{3}$ of the height from the bottom. The amount of the earth thrust (cohesion being neglected) depends on the angle of repose of the soil, but the greatest value that it can ever attain is when the earth becomes saturated with water, and assumes a semi-fluid condition. The pressure, in this case, exerted against the back of the wall is equal to that which would be produced by a fluid of the same specific gravity as the earth, or

$$T = \tfrac{1}{2} w_1 h^2$$

where w_1 = weight of 1 cubic foot of earth

h = height of wall.

But the unit of weight adopted above, was 1 cubic foot of masonry, and if the ratio of the weight of earth to that of masonry be taken as 4 to 5, then

$$T = \tfrac{1}{2} \times \tfrac{4}{5} w h^2 = \tfrac{2}{5} w h^2$$

where

w = weight 1 cubic foot of masonry.

Now this is a definite maximum value for the earth thrust which can under no circumstances be exceeded.

All other thrusts due to different angles of repose are therefore taken as fractions of the pressure produced by earth in a fluid condition, or what may be termed the Mud Thrust.

A scale, the use of which will be hereafter explained, should be constructed from the following table, in which the maximum thrust is taken at 1.000 and all others as fractions of it.

(*See Table on following page.*)

The different values for the thrusts of earth given in the table are calculated from the well known formula

Angle of Repose.	Thrust of Earth.
0°	1.000
5°	0.839
10°	0.704
15°	0.588
20°	0.490
25°	0.405
30°	0.333
35°	0.271
40°	0 217
45°	0.172
50°	0.132
60°	0.071
70°	0.031
80°	0.007
90°	0.000

$$T=\tfrac{1}{2}w_1 h^2 \tan^2.(45°-\tfrac{1}{2}\varphi)$$

where φ=angle of repose.

The use of the scale, as applied to finding the stresses on a dock wall will now be explained.

The stability of the wall depends on the position of the point where the resultant of the earth thrust and the weight of the masonry cuts the base. This point is called the *Center of Resistance* of the base, and its position will vary according to the amount of the earth thrust.

The center of resistance will be furthest away from a line drawn vertically through the center of gravity of the wall, when the earth thrust is greatest, and having once ascertained the position of the center of resistance corresponding to the maximum earth thrust, the scale will enable us to find at once the position of the center of resistance due to a thrust of earth of any required angle of repose.

First then to find the center of resistance for the maximum earth thrust. The usual method is to represent the weight of the wall to some scale, chosen at random, on the line passing vertically through the center of gravity, measuring downwards from a point $\frac{1}{3}$ of the height up. This gives a point, either below or above the base, from which a horizontal line is drawn to represent the earth thrust and the end of it joined with the point $\frac{1}{3}$ of the height up; thus completing the triangle of forces, and the intersection of the resultant with the base is the required center of resistance. If, however, the weight of the wall, instead of being represented to a scale chosen at random, be represented to a scale, such that according to it the weight of the wall=$\frac{1}{3}$ of the height, then the side of the triangle of forces representing the earth thrust will coincide with the base, and if the length of this side, to the same scale be known, then the position of the center of resistance on the base will be found without any further lines of construction. In order to find the required scale, let

A=sectional area of wall in square feet.

h=height of wall

Now by supposition

$$A=\tfrac{1}{3}h$$

for A represents the weight of masonry per foot run

$$\therefore \text{ 1 cubic foot of masonry}=\tfrac{1}{3}.\frac{h}{A}=w$$

But max. earth thrust$=\frac{2}{3}wh^2$

$$\therefore \text{ Max. earth thrust}=\tfrac{2}{15}\times\frac{h^3}{A}$$

which gives the following rule for finding the position of the center of resistance due to earth in a fluid state. Take the height in feet. Cube it and divide it by the number of square feet in the cross section. Multiply the quotient by $\frac{2}{15}$ which will give the distance in feet of the center of resistance from the point in the base lying vertically under the center of gravity of the section. The centers of resistance due to earth possessing different angles of repose are found by dividing up the distance obtained as above, in the same proportion as the divisions of the scale of earth thrusts, previously described. This is effected by parallel lines as shown on the diagram.

In conclusion the new method will be applied to a practical example designed by Mr. A. M. Rendel, M.I.C.E.—the wall of the Victoria Dock Extension, near London, perhaps the largest work in the world which has been constructed in concrete.

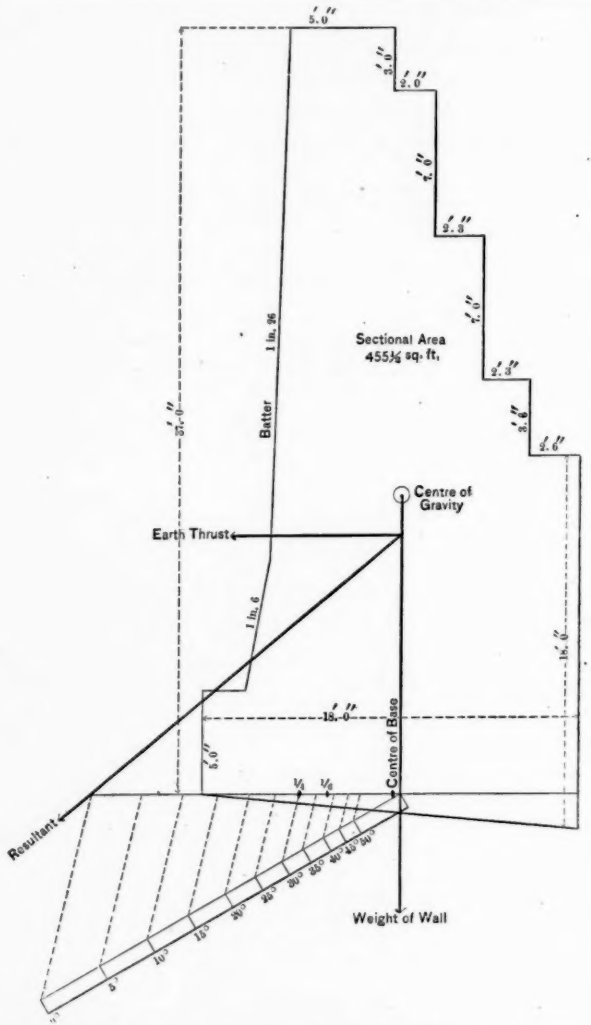
A section of this wall appeared in *Engineering*, March 29, 1878, and is here reproduced.

In this example in question

A=455.5 square feet h=37 feet.

$$\left.\begin{array}{l}\text{Max. earth thrust}\\ \text{measured along base}\\ \text{to required scale}\end{array}\right\}=\frac{2}{15}\times\frac{37^3}{455.5}$$

=14 feet 10 inches



Measure off the distance thus obtained along the base from a point lying vertically under the center of gravity of the section, which gives the position of the center of resistance due to earth in a fluid state.

To find the other centers of resistance for the different angles of repose, place the scale of earth thrusts with one end on the point in the base lying vertically under the center of gravity and inclined at any convenient angle to it.

Draw parallel lines as shown on the diagram.

The result of the investigation is as follows:

Earth in a fluid condition would more than overthrow the wall, and would cause the center of resistance to fall 5′.6″ beyond the toe.

Earth, whose angle of repose is 15° would just overthrow the wall.

Earth, whose angle of repose is 30° would keep the center of resistance with-

in the safe limit of deviation of ¼ the breadth of the base from the center of the base.

Earth, whose angle of repose is 40° would keep the center of resistance within the middle third of the breadth of the base, and there would be no tension in the masonry at any point.

MEANS ADOPTED FOR RANGING THE CENTER LINE OF THE ST. GOTHARD TUNNEL.

By C. DOLEZALEK, Section-Engineer of the St. Gothard Railway.

From "Zeitschrift des Architekten-und Ingenieur Vereins zu Hannover," published by Inst. of Civil Engineers.

The axis of the St. Gothard tunnel is a straight line about 9¼ miles long, with rising gradients of 1 in 172 and 1 in 1,000 respectively from both ends towards its center. At its extremities, viz. in Göschenen and Airolo, observatories were erected, distant 585 and 358 mèters respectively from the tunnel portals, in which were set up the transit instruments previously used in laying out the Mont Cenis tunnel.

The direction of the center line is given from the observatory at night by a lamp placed over that point in it, inside the tunnel, which can be accurately observed directly, its ranging being thence produced by a theodolite as far as the heading permits. A direct observation as far into the tunnel as possible is therefore of the greatest importance, and to obtain this as well as longer station lengths foı the ranging in the interior of the tunnel, the Author devised the contrivances which form the subject of this Paper.

In 1875, to allow the signal to be shown at the right moment to the observer, telegraphic communication was established between the tunnel portal and the observatory, in both of which batteries with Morse's instruments were set up, while, in the unfinished tunnel itself, a wire was joined on by the use of portable field telegraphs.

As petroleum lamps with a bright flame proved far superior to common miners' lamps for signalling at long distances, the Author constructed one with the brilliant-burner ("Rundbrenner") of Schuster and Baer of Berlin, which gave on trial 1.8 time better illumination than the ordinary petroleum lamp. This burner has a double set of pinions moving two half wicks with the greatest regularity, and is screwed on to a large metal vessel having what is called a "double-vase ring." As this allows petroleum to be afterwards poured in without unscrewing the wick-holder, the centering of the lamp (over any station) is not thrown out during the whole period of its use, since the openings in the two rings can be made to coincide or not, at will. The vessel is now leveled on a movable bronze tripod, their centers being made accurately to coincide. This concentric position is in the first instance secured by the maker, but if thrown out at any time, the ring, on which the lamp rests, can be so set by small screws, moving in a circular slit, that the middle of the wick shall be concentric with the tripod, the ring in this case being eccentric to it. This adjustment, however, ought not to be necessary if the lamp is carefully handled. A cylindrical metal mirror is provided to intensify the brilliancy of the flame. This signal lamp surpassed all others in giving far longer station lengths under similar conditions; but it may even yet be advisable to devise apparatus for using the electric light in its place.

To diminish still more the delays and inaccuracies incident on such frequent settings-up of instrument and signal in the tunnel, the Author further constructed a stand applicable to either. It is in two parts, a top plate of metal resting on a large circular one of wood to which three legs are attached. This top plate is separate from the lower one, though capable of being centered accurately with it under or over any required point inside the tunnel, such point being denoted by a notch on an iron cramp,

which is driven into the ground. The weight (nearly 31 lbs.) of the metal plate ensures its steadiness, as its three pointed foot-screws work in small cups let into the wooden plate; by these it is levelled, and when the lamp is placed on it for use, it can be turned round and clamped in any direction.

Every station in the center line was fixed by the mean of eight distinct settings-up of the lamp, by which all level and collimation errors were eliminated from the observations. To deduce this mean readily, the metal plate consists of a bronze plate sliding in a cast-iron frame and provided with a clamp and tangent screw. The center of the bronze plate is given by a notch on either side, while to the two edges of the cast-iron frame strips of gummed paper are affixed, on which each observation is to be recorded by a pencil mark. To the mean of these marks the center of the bronze plate is now set by the notch, and in order that it may necessarily be coincident with that of either lamp or theodolite, as each is successively set up upon the plate, three small grooves radiate from it at angles of 120°, in which are secured the feet of either instrument of whatever size. At the next station the used paper-strips are scraped off, and fresh ones affixed. A plummet and line are attached to the stand for centering purposes.

The advantages claimed for this stand lie in the remarkable speed of "setting-up," in the elimination of all possible errors in the operation, and in the ready insertion of the lamp upon it on the center line; it is also easily carried about the tunnel packed in a chest. The wooden portion is only 1 mèter high besides the round wooden plate of 0.5 mèter outer, and 0.34 mèter inner diameter. Lead weights are attached to the lower parts of the legs to keep them steady if accidentally pushed. Weights above 20 kilogrammes (44 lbs.) should be made up from smaller ones to facilitate their manipulation; and since all the material, instruments, &c., are always forwarded from point to point in the tunnel on trollies, the transport of these lead weights offers no difficulty.

A light transit without vertical and horizontal circles, but with a powerful telescope magnifying thirty times, is advocated for ranging purposes inside the tunnel, and by its use greater rapidity in the work is anticipated.

GAS *VERSUS* ELECTRICITY.

By W. H. PREECE.

From "Nature."

The gas companies are at last awakening to the peculiarity of their position, and gas-shareholders are recovering their confidence in the stability of their property. It is interesting to observe how steadily the shares in all the great gas companies have during the last few weeks been rising, and unless any untoward event occurs there is no reason why in a short time they should not recover the position they so singularly lost in August of last year. Looking dispassionately upon the events that have occurred, it is difficult to understand how such a panic and scare could have arisen. Nothing of any sort or kind has been discovered either in the laws of electricity or in their application to electric lighting to account for it. We know no more of the electric light now than we did in 1862, when as great a display was made in our Exhibition of that year as was made in the French Exhibition of last year. There is no doubt, however, that the enterprise of our neighbors on the other side of the Channel in lighting up so brilliantly one of their grand new streets, produced a sensation that will not easily be forgotten. Englishmen never like to be beaten. We are accustomed to be startled by inventions from the other side of the Atlantic, but we are not accustomed to be beaten either in commercial enterprise or in inventive skill by our neighbors on this side of the Atlantic. Hence,

all of those, whose name is legion, who visited Paris last year, came back with exaggerated ideas of the effect of the electric light in the Avenue de l'Opera, and spread through England a profound opinion of the value of electricity as a means of illumination.

It seems to be forgotten that only three years ago a competitive trial of gas and electricity was made in the clock tower of the Houses of Parliament. Each of these lights was tried for several months, the electric light being a Serrin lamp lit by a Gramme machine; and that, after a very careful examination, gas was successful, was adopted, and is now used by the Office of Works.

Again, it seems to be forgotten that the Elder Brethren of the Trinity House have been experimenting upon this question ever since 1857, and that the results of their experiments have only led to the adoption of the electric light in three of their lighthouses. If the electric light had had the wonderful advantage over gas or oil that its projectors profess for it, surely the governors of such an institution as the Trinity House would have fitted up all the lighthouses upon our coasts with this wonderful light.

The recent experiments, however, have shown both the strength and weakness of the position of the gas companies. Their strength consists in their being in possession of the ground; their weakness consists in their producing only a poor light—and a very poor light—when compared with electricity. But is there any reason why this weakness should continue? Is there any reason why gas should remain such an indifferent light? There is none but that of expense, and expense will not deter people from having a better light if they can only get it. The Phœnix company has taken the question in hand, and has shown in the Waterloo Road what what can be done with gas when the question of expense is not considered. Indeed, it would almost seem, from the experiments that have been made, that the quantity of light to be produced by gas is only a question of the quantity of gas consumed in a given space. There are now burning in the Waterloo Road two brilliant gas lamps, giving a light of 500 candles; and this is greater, in point of fact, than the intensity of the light developed by any one of the electric lights that are now on trial in the thoroughfares of London. There is, however, a defect in gas light which remains to be eradicated, and that is, the color of the light. The one great advantage which the electric light has over gas is, that the electric light, owing to its very high temperature, produces rays of every degree of refrangibility, and therefore, as an illuminating power, it is equal to that of the sun. But gas light, owing to the lowness of its temperature, is deficient in blue rays, and is therefore not so effective in discriminating colors as the electric light.

A very marked advance towards perfection in this direction in gas lighting has been made in the albo-carbon process, by which the gas burnt is enriched with the vapor of naphthaline—a refuse of gas manufacture. This process is being introduced by Mr. Livesey, and, to judge by the experiments that have been shown, it is very promising indeed. The intensity of the light of a gas burner is improved at least five times, and in some experiments, witnessed by the writer, the improvement was as much as twenty times.

The tentative trials that are being made with the electric light in London cannot be said to be very successful.

That at Billingsgate was certainly a fiasco, that on the embankment is very brilliant, but we have yet to learn its cost, and there is no doubt whatever, that the efficiency of the light is very much less than that usually ascribed to the electric light. The trial on the Holborn Viaduct is not a success. The experiment seems to be conducted by some one who is not experienced in the working of electric circuits, for occasionally all the lamps are found extinguished, on other occasions only a portion of them are burning, and frequently they are very dull. It is quite difficult even at the distance of the Post Office, to distinguish the gas from the electric lamp. The same effect is observed on crossing Blackfriars Bridge and looking towards the Houses of Parliament, when there is the slighest mist in the air, and it is quite evident that the electric light has no more—if as much—penetrative power than gas.

A most complete and careful inquiry into the working of the electric light

has been made by Mr. Louis Schwendler for the East Indian Railway Company, and his results are extremely interesting. He has recommended the introduction of the light into certain railway stations where no gas exists, and the system he proposes to use is the Siemens dynamo-machine and one Serrin lamp, and thereby save that waste which the multiplication of the light unquestionably produces. He proposes to distribute this single light by diffusion on a plan originally suggested by the Duke of Sutherland. His investigation has been conducted in a thoroughly scientific spirit, and when his report is published it will be a very valuable addition to our knowledge of the theory of the electric light. It has been shown by the writer that the full effect of the current can only be obtained by one lamp on a short circuit, and that when adding to the lamps by inserting more of them on the same circuit, or on a circuit so that the current is subdivided, the light emitted by each lamp is diminished in the one case by the square, and in the other case by the cube of the number of lamps so inserted. Dr. Siemens maintains also the concentration of the power on one light, but other experimenters are endeavoring to partially multiply the light. For instance, Mr. Rapieff, in the *Times* office, very successfully distributes six lights about the office, and Ladd & Co., with the Wallace form of machine, also distribute six lights over the Liverpool Street Station. Although there is undoubtedly a loss of power in this distribution of the lamps, there may be an advantage in such distribution in cases like printing offices and railway stations. A successful experiment has been made by the British Electric Company, in lighting up some of the stations of the Metropolitan Railway Company, and the India Rubber and Gutta Percha Company have been successful in lighting up the London Bridge station of the London Brighton and South Coast Railway Company. In all these cases we have scarcely emerged from the sphere of experiment. The electric light has not yet been permanently introduced on any large scale. Many are trying it, many are captivated by the brilliancy of the light, and many in their eagerness to keep up with the spirit of the age, are introducing it, as, for instance, the London Stereoscopic Company, and the Messrs. Nichols, the clothiers in Regent Street, where, however, the light does not appear to give very great satisfaction through its fluctuation.

We were led to expect very much from the experiments of Mr. Werdermann, but his attempt to subdivide the light seems to have subsided, for we have heard nothing of it for some time past. Again, we have heard no more of M. Arnaud's discovery, and the accounts that reach us from America of the doings of the Sawyer-Mann light, and of the supposed discoveries of Mr. Edison, are unworthy of attention.

The present state of the electric light question may therefore be said to be a tentative one, and the gas companies are with much enterprise now giving their retort courteous by showing that they are in a position—if the people choose to pay for it—to give quite as powerful a light as the electric light; and, let us hope, before long that it will be quite as perfect. There can be no doubt that the use of electricity for the production of light is a very wasteful as well as a costly process, for the energy that is generated in the machine is not all consumed in the lamp, but is proportionately distributed over the whole circuit. It is, therefore, not utilized only in the place where it is wanted, as in the case of gas. If we are using a certain amount of energy in an electric lamp to light a street, we are wasting as much if not more energy in the street in maintaining the current to produce that light.

There are three points which all electric lights for general purposes should be required to attain. The first is a brilliancy far exceeding that of any known lamp: the second is a durability greater than that which would be required for night operations in England; and the third is absolute steadiness, to enable work to be conducted without affecting the eyes. There is no electric light that has yet been introduced which supplies us with these desiderata.

From the dust of its coal mines France makes annually 700,000 tons of excellent fuel, known here as patent fuel, and Belgium makes 500,000 tons.

RETAINING WALLS.

By Prof. WM. M. THORNTON.

Written for Van Nostrand's Magazine.

1. In the following article it is proposed to give a concise and simplified account of the theory (due to Mr. Lamé) of the Pressure of Earth, and its application to the designing of Retaining Walls by Rankine. Having been heretofore presented as part of a more general theory, it has been needlessly encumbered with mathematical difficulties, and has not been reduced to a form suited to purposes of computation. It is hoped that the mathematical processes here employed will be found intelligible to any one tolerably versed in Algebra, Plane Trigonometry and Elementary Mechanics; and that the results arrived at will prove useful in their application to the purposes of practical design.

2. STRESS.

It is obvious upon the least consideration that the forces between bodies or parts of bodies do not act at definite points but are distributed over surfaces (or volumes) of greater or less extent. When it becomes necessary to consider not the single resultant force but this distributed action we distinguish the latter by the word stress. This stress may vary in amount from point to point; but over a sufficiently small surface it may always be considered constant. The ratio of its total amount to the area of this surface is called the intensity of the stress. The resultant stress must be conceived as applied at the center of this area.

3. INTERNAL STRESS.

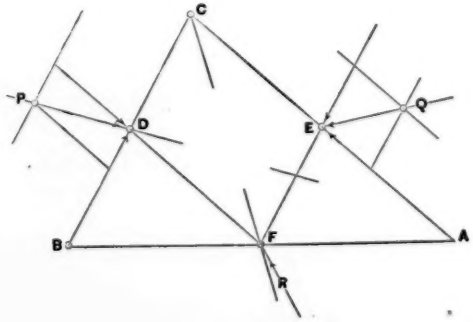
If a body in equilibrio under the action of external forces be anywhere divided by an ideal plane into two parts A,B each of these acts on the other with a certain stress at the plane of division. This is what is called the internal stress on this plane; the inclination of its resultant to the normal to the plane is called the obliquity of the stress; and the stress is a thrust, a shear or a pull as its obliquity is $<=>90°$. This force may, like any other, be resolved into components or compounded with others into resultants. In the questions which we shall have to consider the stresses are all parallel to one plane to which their planes are perpendicular. It will be necessary for us therefore to consider only the particles in a section parallel to this plane.

4. CONJUGATE STRESSES.

The intensities of shear on any two intersecting lines are equal; so that if the stress on a is parallel to b the stress on b is parallel to a.

Proof:—

Consider the elementary triangle ABC, its sides being so small that the stresses on them may be considered uniform, so that the resultants act at the middle points D,E,F. Resolve the stresses P, Q each into components parallel to a, b. The shears P_a, Q_b meet in C. The thrusts P_b, Q_a meet in F. Accordingly to balance R the resultant of P_a, Q_b must



act in CF. Hence $\frac{P_a}{a}=\frac{Q_b}{b}$ which was to be proved. For CD$=\frac{a}{2}$, CE$=\frac{b}{2}$.

The second part of the theorem is obvious. For if the stress on a is parallel to b the common intensity of shear is null. These are called conjugate stresses.

5. PRINCIPAL STRESSES.

Given the components of stress on two orthogonal axes it is required to determine the stress on a third axis through the same origin.

Let p, q denote the normal intensities of stress on Ox, Oy,

Let t denote the common intensity of shear on Ox, Oy,

Let θ denote the slope of the normal to the third axis,

Let σ denote the intensity of stress on it,

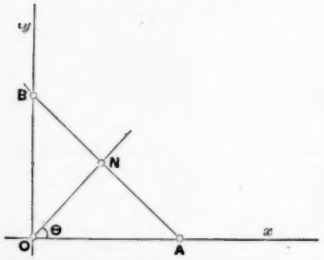
Let ω denote the obliquity of stress on it.

Then the axial components of the stress on AB are

$$X=q.\ OB+t.\ OA,$$
$$Y=p.\ OA+t.\ OB;$$

and the corresponding intensities are

$$x=q\cos.\theta+t\sin.\theta,$$
$$y=p\sin.\theta+t\cos.\theta.$$



The normal and tangential intensities therefore are

$$\nu=x\cos.\theta+y\sin.\theta,$$
$$\tau=x\sin.\theta-y\cos.\theta,$$
$$\therefore \nu=p\sin.^2\theta+2t\sin.\theta\cos.\theta+q\cos.^2\theta,$$
$$\tau=(q-p)\sin.\theta\cos.\theta-t\cos.2\theta.$$

The intensity of shear on the third axis will therefore be null if

$$\tan.2\theta=\frac{2t}{q-p}.$$

This equation determines two axes and only two at right angles to each other on which the stress is wholly normal. These are called the principal axes of stress at O. If they be chosen for co-ordinate axes, the components of stress on an axis whose normal has the slope θ are

$$\nu=p\sin.^2\theta+q\cos.^2\theta,$$
$$\tau=(q-p)\sin.\theta\cos.\theta.$$

The intensity σ and the obliquity ω of the stress on this axis are therefore determined by the equations

$$\sigma\cos.\omega=p\sin.^2\theta+q\cos.^2\theta,$$
$$\sigma\sin.\omega=(q-p)\sin.\theta\cos.\theta.$$

If we eliminate θ between these equations we obtain the quadratic

$$\sigma^2-(p+q)\sigma\cos.\omega+pq=0,$$

which determines the intensities of the two conjugate stresses at O, whose common obliquity is ω. By solving the quadratic we get

$$2\sigma=(p+q)\left\{\cos.\omega\pm\sqrt{\cos.^2\omega-\frac{4pq}{(p+q)^2}}\right\}$$

for the two values of the intensity.

6. MAXIMUM OBLIQUITY.

The expression under the radical in the last equation cannot be negative. Accordingly the maximum obliquity of two conjugate stresses at O is found from the formula

$$\cos.\Omega=\frac{2\sqrt{pq}}{p+q}$$

and the ratio of two conjugate stresses of obliquity ω is

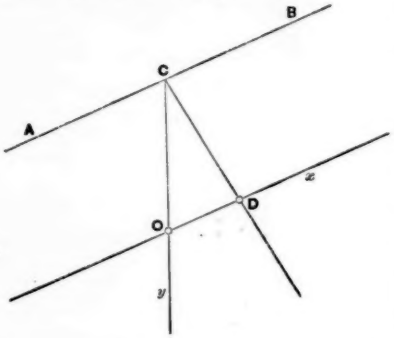
$$\rho=\frac{\cos.\omega-\sqrt{[\cos.^2\omega-\cos.^2\Omega]}}{\cos.\omega+\sqrt{[\cos.^2\omega-\cos.^2\Omega]}}$$

7. FRICTIONAL STABILITY OF LOOSE EARTH.

In such masses of earth as the engineer has to deal with, the only source of stability of which he can take account is the mutual friction of its grains. Adhesion increases this stability; but to an amount which varies so much from extraneous causes that no reliance can be placed on it. The obliquity of stress in such a mass must, therefore, never exceed the angle of repose ψ; that is, Ω cannot be greater than ψ, or the ratio of the less of two conjugate stresses to the greater cannot be less than

$$\rho=\frac{\cos.\omega-\sqrt{(\cos.^2\omega-\cos.^2\phi)}}{\cos.\omega+\sqrt{(\cos.^2\omega-\cos.^2\phi)}}$$

8. In a mass of earth with an indefinite plane for its upper surface, the condition of all sections by perpendicular vertical planes is alike. And at any point O in such a section the stress is vertical on a plane parallel to the upper surface, and equal to the weight of the column OC; that is, the intensity of the stress is $wh\cos.\omega$, where w is the weight of the earth per cubic foot, ω the slope of the upper surface and h the depth. The stress on Oy therefore acts along Ox, and is not less than $\rho wh\cos.\omega$. But this suffices to produce equilibrium at O. It is, therefore, in accordance with Moseley's principle of Least Resistance, the actual intensity of the conjugate stress.



9. PRESSURE OF EARTH ON A VERTICAL PLANE.

If therefore we make OD equal to ρh, the area COD will represent the total pressure on CO, which will be parallel to AB and pass through the center of gravity at COD. That is, the pressure on CO will be

$$P=\tfrac{1}{2}w\rho h^2\cos.\omega$$

and will be applied at two thirds of the depth below C.

10. UPRIGHT RECTANGULAR RETAINING WALL.

The relation between the height and thickness of an upright rectangular retaining wall may now be determined. Either of two methods may be pursued.

Let $t=nh$ denote the thickness of the wall,

Let mw denote the weight of the masonry per cubic foot,

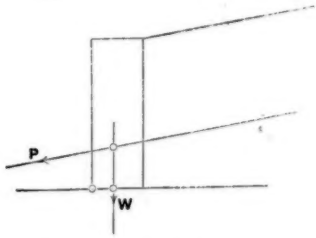
Let s denote the coefficient of safety,

Let qt denote the deviation of center of resistance from center of base,

Let W denote the weight of the wall per foot.

1/. Take moments about the toe of the wall and employ the coefficient of safety. The forces and their lever arms are

$$P=\tfrac{1}{2}w\rho h^2\cos.\omega,\ \tfrac{1}{3}h\cos.\omega-t\sin\omega;$$
$$W=mwht,\qquad \tfrac{1}{2}t;$$
$$\therefore\ \tfrac{1}{2}s\,mwht^2=\tfrac{1}{6}w\rho h^3\cos.^2\omega-\tfrac{1}{2}w\rho h^2 t\sin.\omega\cos.\omega,$$
$$n^2+\frac{\rho s}{m}\sin.\omega\cos.\omega.n=\frac{\rho s}{3m}\cos.^2\omega.$$



2/. Take moments about the center of resistance. The forces and their lever arms are

$$P=\tfrac{1}{2}w\rho h^2\cos.\omega,\ \tfrac{1}{3}h\cos.\omega-\tfrac{1}{2}(2q+1)t\sin.\omega;$$
$$W=mwht,\qquad qt;$$
$$\therefore\ n^2+\frac{2q+1}{4mq}\rho\sin.\omega\cos.\omega.n=\frac{\rho\cos.^2\omega.}{\sigma mq}$$

Of these two methods, the first is a very common practical device for ensuring stability; the second is to be preferred on theoretical grounds. Either of the two quadratics will afford the required value of a.

11. If in the formula which gives the value of ρ we put

$$\sin.x=\frac{\cos.\Phi}{\cos.\omega},$$

we find

$$\rho=\tan.^2\tfrac{1}{2}x;$$

and if in the quadratic

$$n^2+2bn=a^2$$

we put

$$b=a\cot.y,$$

we find

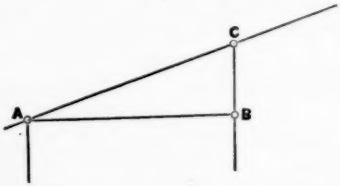
$$n=a\tan.\tfrac{1}{2}y.$$

Employing these substitutions we obtain the following alternative sets of formulæ for designing retaining walls;

1/
$$\begin{cases} \cot.\ y=\sqrt{\frac{3s}{4m}}.\tan.\frac{x}{2}\sin.\ \omega, \\ n=\sqrt{\frac{s}{3m}}.\ \tan\frac{x}{2}\tan.\frac{y}{2}\cos.\omega; \end{cases}$$

and

2/
$$\begin{cases} coty=\frac{2q+1}{8}\sqrt{\frac{6}{mq}}.\tan.\frac{x}{2}\sin.\omega, \\ n=\sqrt{\frac{1}{6mq}}.\ \tan\frac{x}{2}\tan.\frac{y}{2}\cos.\omega \end{cases}$$



12. EXAMPLE.

It is required to design an upright rectangular retaining wall to sustain a mass of earth of practically indefinite extent whose natural slope is 3:2, surface slope 12°30′, the weight of the masonry per cubit being $\frac{5}{4}$ the weight of the earth.

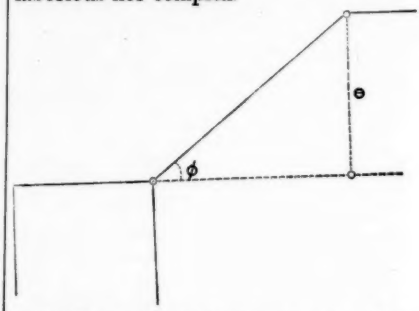
For Φ we have tan. $\Phi=\frac{2}{3}$, whence cos.$^2\Phi=\frac{9}{13}$ and therefore

L cos. Φ	9,92015
L cos. ω	9,98958
L sin. x	9,93057
x	58° 27′ 30″
$\frac{x}{2}$	29° 13′ 45″

In applying the other formulae we assume $s=2$ for the first set, $q=\frac{3}{8}$ for the second.

L tan. $\frac{x}{2}$	9,74784	9,74784
L sin. ω	9,33534	9,33534
	0,00959	$\bar{1}$,89356
L cot. y	9,12377	8,97674
y	82° 26′ 34″	84° 35′ 7″
$\frac{1}{2}y$	41° 13′ 17″	42° 17′ 34″
L tan. $\frac{x}{2}$	9,74784	9,74784
L tan. $\frac{y}{2}$	9,94255	9,95890
L cos. ω	9,98958	9,98958
	$\bar{1}$,86350	$\bar{1}$,77546
Ln	$\bar{1}$,54347	$\bar{1}$,47178
n	0,34952	0,29633

That is, the first method gives $n=0.35$; the second $n=0,30$. The first value happens to be exactly that recommended by Trautwine [P.B. 331]; the second that recommended by Molesworth [P. B. 17]. The second is doubled sufficient. The calculations as will be seen are neither laborious nor complex.



13. The coefficient of safety of a given retaining wall is found from the relation (10.1) which gives

$$s=\frac{6mn^2}{\tan.^2\frac{1}{2}x\sin.2\omega[\cot.\omega-3n]}$$

Thus using the example of (12) we have

cot. ω	4,5107
3 n	0,9
	3,6107
	0,55759
2 L tan. $\frac{1}{2}x$	9,49568
L sin. 2 ω	9,62595
	$\bar{1}$,67922
L 6 m n^2	$\bar{1}$,82930
L s	0,15012

whence $s=1,413$. In this way the engineer can always judge whether the proposed value of n is sufficient.

14. The inclination i of the resultant to the vertical is given by the formula

$$\frac{\cos.(\omega+i)}{\sin.i}=\frac{W}{P},$$

or

$$\cot.i=\tan.\omega+\frac{2mn}{\tan.^2\frac{1}{2}x\cos.^2\omega}$$

Thus using the same example we have

L tan. $\frac{1}{2}x$	9,74784
L cos. ω	9,98958
	$\bar{1}$,73742
	$\bar{1}$,47484
L 2 m n	$\bar{1}$,87506
	0,40022
	2,5132
tan. ω	0,2217
cot. i	2,7349

whence $i=20°$ which is less than the angle of repose of the masonry on its foundation. Should the result exceed

this angle, then the base of the wall must be tilted back at an angle greater than the excess.

15. RESUME.

Let Φ=the natural slope,
" ω " surface slope,
" m " density of masonry [earth=1],
" q " deviation of center of resistance,
" n " ratio of thickness to height,
" s " coefficient of safety,
" i " obliquity of pressure on base.

$$\sin. x=\frac{\cos. \Phi}{\cos. \omega}$$

$$\cot. y=\frac{2q+1}{8}\sqrt{\frac{6}{mq}}\tan. \tfrac{1}{2}x \sin. \omega$$

$$n=\sqrt{\frac{1}{6mq}}\tan. \tfrac{1}{2}x \tan. \tfrac{1}{2}y \cos. \omega$$

$$s=\frac{6mn^2}{\tan.^2 \tfrac{1}{2}x \sin. 2\omega\,[\cot. \omega-3n]}$$

$$\cot. i=\tan. \omega+\frac{2mn}{\tan.^2 \tfrac{1}{2}x \cos.^2 \omega}$$

16. TABLE.

The following table is computed from the above formulae with the date

$$\tan. \Phi=\tfrac{2}{3},\ m=\tfrac{5}{4},\ q=0{,}3;$$

$$\therefore \frac{2q+1}{8}\sqrt{\frac{6}{mq}}=0{,}8$$

$$\sqrt{\frac{1}{6mq}}=\tfrac{2}{3}.$$

ω	n	s	i	$\frac{1}{2}x$	L tan. $\frac{1}{2}x$
0°	0,357	1,669	17° 48′	28° 9′19″	9,72851
2	0,352	1,683	17° 51′	28 10 50	9,72897
4	0,347	1,693	17° 57′	28 15 38	9,73043
6	0,343	1,711	18° 2′	28 23 45	9,73288
8	0,339	1,728	18° 8′	28 34 56	9,73625
10	0,335	1,741	18° 17′	28 49 49	9,74072
12	0 332	1,762	18° 25′	29 8 26	9,74626
14	0,328	1,778	18° 38′	29 31 13	9,75300
16	0,3 5	1,788	18° 51′	29 58 30	9,76100
18	0,323	1,822	19° 3′	30 30 52	9,77040
20	0,321	1,846	19° 20′	31 9 10	9,78139
22	0,320	1,926	19° 38′	31 54 30	9,79424
24	0,319	1,917	20° 1′	32 48 30	9,80933
26	0,319	1,963	20° 30′	33 53 24	9,82719
28	0,320	2,018	21° 7′	35 13 36	9,84888
30	0,323	2,098	21° 57′	36 57 0	9,87633
32	0,330	2,202	23° 18′	39 25 40	9,91499
Φ	0,369	2,802	26° 34′	45° 0′ 0″	10,00000

It contains the values of $\frac{1}{2}x$, L tan. $\frac{1}{2}x$, n, s, i for equidistant values of ω. For any other values the corresponding results may be found by an easy interpolation.

For any other value of m the value of n, which corresponds to an equal value of s, may be found with sufficient accuracy for practice from the formula

$$\frac{n'}{n}=\sqrt{\frac{m}{m'}}$$

Thus if $m'=\frac{3}{2}$ and $m=\frac{5}{4}$ as above

$$n'=0{,}913\ n.$$

The value of Φ adopted is that which is almost universally found for ordinary earth work in dry earth.

17. When the earth slopes to the outer edge of the wall it is recommended to measure the height to c and use this to compute the thickness. Or, what amounts to the same thing, to take n from the table and make the ratio of height to thickness

$$\frac{1}{n}-\tan. \omega$$

This gives a very slight excess of stability.

18. SURCHARGED WALLS.

A wall is said to be surcharged when the backing slopes away from it at the angle of repose to a certain height called the height of surcharge after which it is horizontal. The thickness of the wall is thus somewhat less than it would have to be if the slope were indefinite. According to Rankine we may put

$$n=n_{\Phi}-\frac{n_{\Phi}-n_0}{1+2\theta}$$

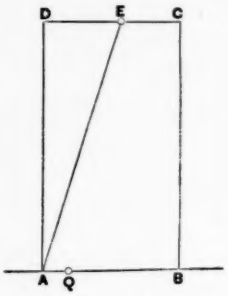
where θ is the height of surcharge, the height of the wall being taken as 1. With the data already used this formula becomes

$$1000n=369-\frac{12}{1+2\theta}$$

The limitation of height will not decrease the thickness therefore by as much as one thousandth part unless $\theta<5\frac{1}{2}$.

19. BATTERING-FACED WALLS.

An upright rectangular wall ABCD with center of resistance at Q may be changed



into a battering wall ABCE of equal stability by making DE=3AQ. For the center of gravity of ADE being vertically over Q, this change does not affect the moment of stability. Thus with the same data as before AQ=0,2n; DE=0,6n; and the batter is 1:0,6n.

This change obviously increases the obliquity of the pressure on the base and to guard against sliding we should give the batter a cant backwards.

Any batter less than this may, of of course, be used.

GAS ILLUMINATION.

By Dr. WILLIAM WALLACE, F. R. S. E.

From "Journal of the Society of Arts."

It is impossible to discuss the subject of gas illumination at the present time without referring to the electric light, which many authorities affirm is destined to be the light of the future. If this is so, it might naturally be inferred by those who have only a slight knowledge of the subject, that it is only wasting energy to devote time and study to the improvement of gas lighting, since it must soon be superseded by the more brilliant light obtained from electricity. I have given this matter some attention, and I must say that I have no fear that gas interests will suffer in consequence of the introduction of the electric light for many, many years, if at all. The mere fact that light can be obtained by passing a powerful current of voltaic electricity between carbon points, dates back to a time when gas lighting itself was only in its infancy; and it is now nearly 30 years since the apparatus was so far perfected that the distance between the carbon points was worked automatically; and the improvements recently introduced, if we accept the Jablochkoff candle, and the imperfect arc formed in the Werderman arrangement, have been directed more to the effective production of electricity by mechanical power than to the light itself. Turning over an old periodical a few days since, I came upon a paragraph which I read with some interest, in which it stated that a French *savan* had discovered a plan by which the unsteadiness of the electric light was removed. The date of this announcement is 1853—a quarter of a century since—and even now we are scarcely in a position to say that the unsteadiness of the light has been overcome. The fact is, that we have still a long period of experiment and study before us in regard to lighting by electricity, and although the march of improvement in science is now extremely rapid, I scarcely hope to live long enough to see electricity take the place of gas in the lighting of ordinary dwelling-houses. But even if I am in error in supposing that the enormous difficulties will not immediately be overcome, there is still little, if any, cause for alarm on the part of holders of gas stocks, since, even at the worst, gas is certain to be used side by side with electricity, as long as coal can be got to produce it. The fears entertained recently by shareholders of gas companies, remind me of the beginning of railway engineering, when it was asserted that if railways were allowed to be made there would no longer be any use for horses, and the valuable breeds of the animal in this country would be allowed to die out. We all know that the result was entirely the other way; the railways increased the demand for horses, and they became more valuable and more numerous than ever. Then, again, it was supposed that when gas was used for the lighting of towns the manufacture of candles would cease,

but what is the fact? More candles are made now than ever there were before, and, what is very much to the purpose in connection with my subject to-night, the greatest improvements in the manufacture of candles were made after the gas manufacture was fairly established. Even within my own recollection, candles were burned which required constant snuffing, and so late as 30 years ago artistic designs for snuffers and snuffer-trays were published in art journals. If electricity supplants gas for public lighting, as I believe it may to some extent, it is all the more necessary that we should strive to get more light out of gas, either by improvements in the mode of manufacture, or by better means of burning it, or both, and I am very sanguine that gas lighting during the next 30 years will be developed to an extent of which we can at present form no adequate idea. We have seen some improvements in gas lighting already. What was at one time 12-candle gas, tested by the primitive Argand, became 14-candle gas with the Sugg-Letheby burner, and 16-candle gas with Sugg's London Argand; and all this without sensibly changing the quality of the gas, and, consequently, without conferring any benefit on the public. A real and substantial improvement in gas lighting would be one which would enable the public to get, in ordinary domestic life, something approaching to the illuminating power declared as the result of the official tests, and the object of this paper is to show what has been done up to the present time in this direction. Before passing on to my subject, however, I wish to make just one remark. If the the production of gas is sensibly decreased, the value of the by-products will proportionately rise, the demand for benzole, anthracen, tar oils, pitch, and ammonia, will continue; and, if the quantity produced becomes less, the value of these important articles will undoubtedly increase.

Coal gas is a cheap source of light, the only real competitor in this respect being paraffin oil. The following table gives the comparative values, based on what may be accepted as average prices, although some of them may not be exactly correct at the present time:

Cannel gas, 30 candles, at	4s.	2d.	₱ 1,000 c.ft.	1
Common gas, 16 candles	3s.		"	1¼
Paraffin oil	1s.	6d.	₱ gallon	1¼
Colza oil in moderator lamps	4s	6d.	"	7
Stearine candles		10d.	₱ pound	27
Tallow "		8d.	"	29
Paraffin "	1s.	6d.	"	31
Sperm "	1s.	6d.	"	36
Wax "	2s.	6d.	"	72

In these comparisons, it is but fair to say, the gas is calculated as giving the light obtained when burned in the best known manner, as in the official tests of the gas examiners of the towns where the respective qualities of gas are made.

It will be well to indicate, in a few words, the principle involved in the testing of various gas flames and other sources of light. If a flame of any kind is held at any distance, say a yard, from a screen, in which an opening is made one foot square, and a second screen is placed at the distance of two yards, there will be thrown upon the latter a square figure, which, on examination, will be found to measure exactly 2 feet, and which has, therefore, an area of 4 square feet. If the second screen is moved to 3 yards, the illuminated portion will measure 3 feet square, representing an area of 9 square feet; at 4 yards it will measure 4 feet, giving an area of 16 square feet. We thus see that the space covered by the light increases in proportion to the square of the distance, while the intensity of light decreases in a corresponding degree. To put the matter more clearly to those who have not studied the subject—a flame at a given distance, say a yard, illuminates a given space, say a foot square, but at four times the distance the illuminating effect is diffused over 16 times the area, or 16 square feet, consequently any single square foot at this distance gets only one-sixteenth part of the whole quantity.

I do not propose to enter into any details regarding photometers, all of which are based upon this principle, but I may explain the mode of testing by a simple illustration. I have a space of 100 inches, with a candle at one end, and a gas flame which I wish to test at the other. I have a greased disc moving freely between the two, and by a little practice I can place it in a position in which the two sides are equally illumin-

ated. I now measure the difference between the candle and the disc, and find it to be 20 inches, while that between the gas flame and the disc is 80 inches; the square of 20 is 400, and of 80, 6,400, and the one divided by the other, gives 16 candles as the illuminating power of the gas flame. In practice, the photometer is divided so as to give the illuminating power by direct observation, and many details require to be minutely attended to in order to obtain reliable results.

We are in the habit of talking of certain qualities of gas—16 candles in London, 15 in Birmingham, 14 in Newcastle, 26 in Glasgow, 30 in Edinburgh, but these are not the values of the gas as burned in our houses, warehouses, and shops, but as burned in the manner calculated to give the highest illuminating power. These figures show the possibility of gas illumination, and represent the goal toward which we should strive. I freely admit that it is impracticable, not to say impossible, to obtain in the every-day practice of common life, results as good as those got by means of appliances the most perfect for developing the full photogenic value of the gas, but still a great deal may be done to decrease the reckless waste of light that is constantly going on. I have no hesitation in saying that from 12 to 14 candle power might be obtained in every-day life from what is called 16-candle gas. We stand in a similar position with regard to various forces employed for practical purposes. The engineer calculates the power that should be obtained by the falling of a given weight of water through a given space, but the practical result obtained in a water-wheel always falls far short of the theoretical quantity. In like manner, the force obtained by the combustion of one pound of coal in the boiler of a steam-engine is greatly less than the calculated figure. Still, mechanicians struggle on to obtain better results, and we are constantly improving. Some of the most recent forms of reaction engines show an immense improvement on the water wheels formerly in use, while, in regard to steam power, we have, in the performance of the best descriptions of compound engines, an approach to theory which was formerly deemed impossible of attainment. Such improvements represent so much money saved to the country; and it is equally the case with gas, but with this addition, that a decreased consumption, with the same amount of light, would give increased healthfulness to our dwellings, where the products of the combustion of gas constitute an evil of no small magnitude. It has been said that the man who causes two blades of grass to grow where only one grew before, is a benefactor to his country, and some honor is due also to him who enables us, by improvements in the steam-engine, to get out of one pound of coal the power which formerly required the combustion of two pounds, or, who teaches us how to obtain from one cubic foot of gas the illuminating value for which two feet had previously been expended.

When a porcelain slab is brought over a gas flame a deposit of carbon occurs: the particles exist in the flame, and the contact of the cold slab causes their instant deposition. A similar effect is produced by a current of cold air impinging upon a flame, a portion of which is thus cooled down below the temperature necessary for the combustion of the carbon, and the flame thus exposed to the draught smokes, that is, the finely divided particles of carbon pass into the air unconsumed. In ordinary circumstances, the carbon is consumed in the upper portion of the flame, and if the jet be a good one, and the pressure of gas not too low, no smoke is produced. In the Bunsen burner the gas is mixed with air sufficient to prevent the separation of the carbon, and hence we have a flame which is valueless as a source of light, but convenient for the application of heat. The solid particles of carbon in an ordinary gas flame result from the decomposition of the olefines and other compounds rich in carbon, which are readily decomposed by the action of heat. The same thing occurs if coal gas be passed through a glass tube heated to redness; in this case a deposit of carbon in the interior of the tube occurs at the point where the heat is applied. In gas works a similar effect is produced by the heating of the impure gas in the retorts, in which a deposit of carbon, sometimes three or four inches in thickness, is formed. This carbon was formerly used for the rods or pencils employed in producing the electric light. The presence of the particles of carbon in

a flame renders it opaque, and the degree of opacity varies with the illuminating power. At the bottom of a flat flame, where the oxygen is in excess, the transparency is such that a printed paper may be read through it as if no obstruction intervened, but the upper part almost entirely conceals the printing. The intensity of light depends partly upon the quantity of the carbon particles, and partly upon the heat of the flame by which the carbon is brought up to a greater or less degree of incandescence. Professor Frankland has shown that the light is not entirely due to the separated carbon, and that certain chemical compounds—gases or vapors—from which carbon does not separate by the action of heat, are capable, under some conditions, of giving luminous flames when burned in air. For all practical purposes, however, the original proposition of Davy may be accepted, that the light is radiated from highly heated particles of solid carbon. When air is supplied in excess to a flame, as when the gas escapes through a fine jet at a high pressure, there is little separated carbon, the flame is transparent, or nearly so, and there is very little luminosity. On the other hand, when the flame is large and sluggish, and the air in contact with it is insufficient, the solid carbon is in excess, and a part of it escapes unburnt, giving rise to a smoky flame, in which also the luminosity may be low. What we have to strive after in order to obtain the greatest possible "duty," as mechanical engineers would call it, from gas, is to burn it so as to have the flame as hot as possible, and as near the smoking point as is consistent with the perfect consumption of the carbon in the upper part of the flame. In few words, the whole science of gas lighting is the obtaining a bright flame without smoke. It was at one time time accepted as an axiom that economy in gas lighting could only be obtained by the use of large burners, and that in small jets the contact of air was necessarily so complete that only a feeble light could be obtained. But this is only partially true; more precise and extended experiments have shown that the luminosity depends not so much upon the quantity of gas as upon the conditions under which it is burned. In the case of flat-flame burners, the most essential element is pressure, a high initial velocity giving a low illuminating power, and *vice versa*. I may give a few illustrations from my own experiments—the gas used being of 26 candles illuminating power for five cubic feet per hour. In all the instances I am about to quote Bray's ordinary union jets were used. The gas gave the most unfavorable result when the smallest burner of the series, No. 0, was used under comparatively high pressure—1½ inches—two cubic feet per hour gave an illuminatng effect of 3.21 candles, or calculated to the standard of five cubic feet per hour, eight candles. The best result, on the other hand, was obtained with a No. 8 burner at one inch pressure, when 7.1 cubic feet gave an illuminating effect of 40.63 candles, or for 5 cubic feet, 28.6 candles. Here is a striking contrast, the same gas giving at one time 8-candle power for five feet an hour, at another 28.6, the jets being respectively the smallest and the largest of the series of nine. But let us now take the same quantity of gas under varied conditions of pressure, and we shall see even here very marked differences. Three cubic feet burned at half-inch pressure, and calculated to five feet per hour, gave 25 candles; at one inch pressure, 19 candles; and at 1½ inch, 12½ candles. Here we have the effect simply of pressure, which, in the case of flat-flame burners, is of paramount importance. When common gas is used, the effect of pressure is even more remarkable, the varieties being such that in some cases less than one fourth of the possible amount of light producible is really obtained.

A remarkable effect is obtained with a mixture of cannel gas with about twice its bulk of air. At a low pressure, in an Argand jet with large holes, it gives a fairly luminous flame, while at a high pressure (3 or 4 inches), although the quantity of gas consumed is three times as great, the flame is almost totally non-luminous, and has a greenish tint. The gas used somewhat extensively in the United States, made by saturating air with petroleum spirit, requires to be burned at a pressure not exceeding 0.1 of an inch, which can be obtained only with an Argand with very large holes, or a bat's-wing of peculiar construction, called the American regulating bat's-wing. At ordinary pressures, such as are used for coal gas, there is scarcely any light, and

the flame keeps about a $\frac{1}{4}$ inch or more above the burner.

It is not only on the score of economy that it is desirable to burn gas in such a manner as to afford the greatest possible amount of light. The burning of a moderate-sized jet of gas produces as much carbonic anhydride as the breathing of two grown-up men; and as, in an ordinary apartment, we have usually from three to six of these, the air becomes vitiated with remarkable rapidity. It is, therefore, desirable, in relation to health, to obtain the illumination we require with the least possible expenditure of gas. The sulphur in gas is a very serious drawback to its use. In burning, it is no doubt formed chiefly, if not entirely, into sulphurous anhydride; but it is soon converted into sulphuric acid, which attacks with avidity all the more readily destructible articles in the apartment. So far back as 40 years since, the effects of the sulphuric acid arising from the combustion of gas upon the binding of books and many articles of furniture were noted; and recent experiments have shown that leather, paper, etc., in ill-ventilated apartments, exposed to the emanations from burning gas for a series of years, contain large quantities of sulphuric acid.

There are several qualities of gas in use in this country. The best may be described as Scotch cannel gas, as it is made only in Scotland, where the illuminating power varies from 24 to 30 candles for 5 cubic feet per hour consumed in a union or fish-tail jet: the average may be fairly stated as 26 candles. In London a cannel gas is used in small proportion, the illuminating power of which is about 23 candles; and in Liverpool, Manchester, Carlisle, and probably some other towns, an intermediate gas is manufactured, the illuminating power of which is about 20 candles. The common gas in London, and most other English and Irish towns, has an illuminating power of 14 to 16 candles. In the case of cannel gas, the standard is found by testing the gas by a union jet consuming 5 cubic feet, at a pressure of 0.5 of an inch, while the common gas is tested by Sugg's "London" Argand, consuming 5 cubic feet per hour, at a pressure of about 0.05 of an inch.

The burners at present in use may be divided into the four following classes:—1. Cockspur, or rat-tail. 2. Union, or fish-tail. 3. Bat's-wing. 4. Argand. Of each of these there are a number of modifications.

The cockspur, or rat-tail burner, is the simplest possible form of gas-jet, and it was at one time the only one used for burning gas. It may be made by simply drawing out a piece of glass tube and breaking off the point so as to leave an orifice having a diameter of 1 millimetre or less; but it is usually constructed of cast iron, which is drilled as wide as possible from the bottom, leaving only a thin shell, which is then bored with a fine drill. Two sizes of these were tested, No. 1 having an orifice of about 0.6, and No. 2 of about 0.75 millimetre. These jets are used in Glasgow for lighting common stairs, and the larger sizes were formerly employed for street lamps, but are now discarded in favor of union jets. The following are the results with 26-candle gas:—

No. of Burner.	Pressure in Inches.	Length of Flame in Inches.	Gas per hour.	Illuminating Power in Standard Candles.	Illuminating Power of Five Cubic Feet per hour in Candles.
1	0.5	2	0.45	0.89	9.9
1	1.0	$3\frac{1}{2}$	0.60	1.69	14.1
1	1.5	$4\frac{3}{4}$	0.90	2.40	13.3
2	0.5	$3\frac{7}{8}$	0.80	2.49	15.6
2	1.0	$5\frac{1}{2}$	1.13	3.55	15.7
2	1.5	$7\frac{1}{4}$	1.45	4.53	15.6

These figures show that even with the larger jets no more than 60 per cent. of the real value of the gas can be obtained. I have tried various modified forms of the jet, some having "adamas" tips and contracted at the bottom, or otherwise obstructed so as to diminish the pressure at the point of ignition, but they did not show any marked superiority over those referred to above.

When two rat-tail jets are held at a right angle to one another, the lights coalesce and form a flat sheet of flame. When this discovery was first made, two burners were fitted up in this way, but soon a single burner was contrived which combined the two, and hence was called a "union" jet; it is also known as a fish-tail, from the resemblance of the flame to the tail of a fish. It is a short cylindrical tube with a flat top, in which the two

orifices are drilled at about 90 degrees to one another, and meeting in the center. The union jet is much improved by substituting for the metal top porcelain or stoneware, the principal advantage gained being that the orifices remain clean and constant in size, while those of iron gradually rust up and require to be frequently cleaned in order to give a satisfactory light, and are consequently enlarged. Some fish-tail burners are made entirely of a kind of stoneware or of steatite, but these are troublesome to remove when they get broken. The best form of burner is that with a brass body and porcelain top. Such burners are made by Leoni of London, Bray of Leeds, and other makers, but usually with some means of reducing the pressure. The fish-tail burner is not suited for burning at a high pressure, under which the two flames refuse to spread out into a flat sheet, but form an irregular flame, at the same time emitting a most disagreeable hissing or blowing sound. This effect may also result from other causes, such as a sharp bend in the gas-supply tube, a speck of dust in one of the orifices of the burner, or, in fact, anything that disturbs the even and quiet flow of the gas. One singular example of this is the following:—If a union jet is burning five cubic feet of gas at 0.5 inch pressure, and a portion of gas is led away by means of a tube inserted a few inches below the flame, although diminished in volume, it immediately begins to blow.

In testing flat flames, the custom has invariably been to present the flat side to the disc of the photometer; but, although the results so obtained are satisfactory in comparing one flat flame with another, they cannot fairly be compared with rat-tail or Argand flames, which give an equal light all round. The edge of a bat flame gives considerably less light than the side, but the difference between the two depends very much upon the richness of the gas, or, in other words, the opacity of the flame. The following example may be given:—A union jet, consuming five cubic feet of cannel gas, at 0.5 inch pressure, gave a light of 27 candles when tested in the ordinary manner with the flat side towards the photometer disc; but the edge gave only 23 candles, and when rotated, so as to give the flame in every position, the average result was as nearly as possible 26 candles, showing that the ordinary test gave one candle too much, or nearly 4 per cent. In the case of paraffine flat-flame lamps, the difference between the front of the flame and the average all round varies from 4 to 10 per cent. In the latter case, the flame is intensely opaque, and of a deep yellow color. All the figures given in this paper refer to the flat side of the flame, and this must be borne in mind in comparing flat with round flames.

The following Table gives the results obtained with Bray's union jets, without obstruction to retard the flow of gas and reduce its pressure. Gas by ordinary test, 26 candles:—

No. of Burner.	At 0.5 Inch Pressure.			At 1.0 Inch Pressure.			At 1.5 Inch Pressure.		
	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
0	1.15	1.96	8.52	1.55	2.35	7.60	1.80	2.45	6.80
1	1.45	3.77	13.00	2.15	5.28	12.28	2.85	5.47	9.60
2	1.70	4.85	14.27	2.50	6.74	13 48	3.10	7.62	12.30
3	2.35	8.98	19.10	3.40	11.73	17.25	4.30	13.67	15.90
4	2.85	11.97	21.00	4.05	15.44	19.06	5.00	16.62	16.62
5	3.25	13.84	21.30	4.50	18.78	20.87	5.55	21.90	19.73
6	4.10	19.60	23.90	5.70	25.60	22.46	—	Gas blows.	—
7	4.75	24 76	26.00	—	Gas blows.	—	—	"	—
8	5.00	26.00	26.00	—	"	—	—	"	—

This Table gives instructive information as to the effects of mass or quantity of gas and of pressure. As regards mass, we see that at the same pressure the light afforded by 5 cubic feet of gas per hour varies from 8½ to 26 candles, according to the quantity burned, the lowest result being obtained with about 1 cubic foot per hour, and the highest with 5 cubic feet. This last result—*i.e.*, 26 candles for 5 cubic feet of gas per hour, burned in a union jet at 0.5-inch pressure, is taken as the standard of comparison in all the experiments on cannel gas. The ratio of illuminating power to quantity is nearly the same at higher pressures, and there is no difficulty in deducing the general law that the value in illuminating effect per cubic foot of gas increases with the mass of the flame.

The effects of pressure are not less striking, and might have been more so had the gas been tested at lower pressures than 0.5-inch and higher than 1.5-inch. The results obtained with a jet consuming 5 cubic feet per hour gave 26 candles at the low pressure, and only 16.6 at 1.5-inch, showing a loss of lighting power amounting to about 36 per cent.; 3 feet per hour, calculated to 5 feet, gave at the low pressure 21 candles, at the high pressure 12.3 candles; the burner being a No. 4 in the one case, and a No. 2 in the other. The medium pressure gave results intermediate between these. At the higher pressures some of the larger sized burners became useless, as already explained.

As, in practice, it is found impossible to distribute gas at a pressure of less than 12 or 15-10ths of an inch of water, various contrivances for breaking the force of the gas have been invented. Among union jets of this kind, the simplest, perhaps, is that of Leoni, consisting of a brass and an iron tube, which fit into one another, and between which a thin film of cotton wool is placed. This is a very good burner, but it cannot be depended upon for delivering exact quantities of gas. Bray has constructed a very good burner, similar to those already mentioned, but having a double ply of cotton cloth stretched across a metal ring placed in the tube, in order to relieve the pressure. The same manufacturer has more recently invented another burner, in which the reduction of pressure is attained by passing the gas through an orifice in a porcelain plate cemented into the lower part of the burner. He calls these "Special" burners, and they are of two kinds—one intended for general use, and the other for street lamps, in which the orifices are somewhat smaller, and in which, consequently, the pressure is further reduced. Morley's patent burner is of brass and vase-shaped, with a porcelain top, and at the bottom one or two small orifices in the metal for admitting the gas. Williamson's jet is similar in principle, but more complicated in construction. Da Costa's burner consists of a hollow vase stuffed with iron turnings, into which an ordinary iron union jet is screwed. There are others, but all have the same object in view, and the simpler and cheaper burners, such as Leoni's and Bray's, accomplish it as successfully as those of more complicated construction, and these have, therefore, been selected for a series of comparative trials, all being made with 26-candle gas. Some of the burners referred to are called regulators, but this is a mere name, for it is obvious that they merely obstruct the flow of the gas, the quantity delivered rising as the pressure is increased. In Bray's "Special" burner the two holes forming the "union" jet are placed at an angle of 120°.

In both series of the "Special" burners, in which the pressure is much reduced, the best results are obtained at 1-inch pressure, while, at .5 inch, the flames are sluggish, and, in some cases, show a tendency to smoke. This is not the case, however, when common gas is used.

Mr. Holdsworth, of Bradford, has introduced a simple arrangement which he calls a gas-feeder, which has been adopted rather extensively in the manufacturing towns of Yorkshire. It is simply a little wedge-shaped piece of lead pierced in the centre with a hole, the area of which is less than that of the holes in the burner, and this is fixed in the gas pipe, several inches from the burner. Several sizes are made, to suit varying circumstances of local pressure, as well as different sizes of burners, and if fitted up by an intelligent workman, they accomplish the end in view very successfully.

Many years ago, Mr. Scholl, of London, adopted the system of placing a

BRAY'S "REGULATOR" UNION JETS FOR CANNEL GAS.

No. of Burner	At 0.5 Inch Pressure.			At 1.0 Inch Pressure.			At 1.5 Inch Pressure.		
	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
0	1.00	2.72	13.6	1.50	3.13	10.4	2.00	3.21	8.0
1	1.15	3.75	16 3	1.80	4.30	11.9	2.45	4.40	9.0
2	1.50	5.63	18.7	2.30	7.25	15.8	3.00	7.60	12.6
3	1.80	7.97	22.1	2.75	10.11	18 4	3.55	11.37	16.0
4	2.40	11.26	23.4	3.60	15.21	21.1	4.30	15.32	17.8
5	2.60	12.76	24.5	4.35	20.40	23.4	5.10	22.19	21.7
6	3.15	15.95	25.3	4.95	25.42	25.7	5.80	27.51	23.8
7	3.80	20.07	26.4	6.05	32.75	27 1	..	gas blows.	..
8	4.70	24.76	26.3	7.10	40.63	28.6	..	"	..

BRAY'S "SPECIAL" UNION JETS FOR GENERAL USE.

No. of Burner	At 0.5 Inch Pressure.			At 1.0 Inch Pressure.			At 1.5 Inch Pressure.		
	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
0	1.44	5.51	19.13	2.16	9 22	21.34	2.59	10.80	20.85
1	1.55	6.11	19.71	2.36	10.33	21.88	2.87	12.00	20.91
2	1.86	7.50	20.16	2.76	12.38	22.43	3 36	14.51	21.59
3	2.10	8.90	21.19	3.10	14.27	23.01	3.75	17.29	23.11
4	2.44	10.94	22.42	3.62	17.69	24.43	4.41	20.83	23.60
5	2.71	13.39	24.70	4.13	21.13	25.58	5.16	26.17	25.36
6	3.12	15.42	24.71	4.76	24.40	25.63	5.71	28.66	25.09
7	3.63	18.43	25 39	5.51	28.65	26.00	6.70	34.33	25.62
8	4.28	22.26	26.00	6.39	34.37	26 89	7.92	40.67	25.67

BRAY'S "SPECIAL" UNION JETS FOR STREET LAMPS.

No. of Burner	At 0.5 Inch Pressure.			At 1.5 Inch Pressure.			At 1.0 Inch Pressure.		
	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
0	1.30	4.85	18.65	1.96	8.22	20.97	2.34	9.70	20.73
1	1.46	6.04	20.68	2.21	9.57	21.65	2.63	11.45	21.77
2	1.73	7.28	21.04	2.56	12.00	23.44	3.01	14.86	24.68
3	2.07	9.36	22.61	3.00	14.64	24.40	3.57	17 63	24.69
4	2.24	10.73	23.95	3.33	16.57	24.88	4.05	20.39	25.17
5	2 68	13.15	24.53	4 08	21.17	25.94	4.85	25.67	26.46
6	2.97	14.77	24.86	4.45	23.73	26.66	5.37	28.87	26.88
7	3.44	17.37	25.25	5.31	28.36	26.61	6.43	34.32	26.69
8	3.84	19.21	25.01	5.92	31.22	26.37	7.23	37.32	25.81

small piece of platinum between the two orifices of the union jet, the result being that the initial velocity with which the gas escapes is spent by striking against this plate, and the gas ascends in a somewhat sluggish flame, which, in the case of cannel gas, has a tendency to smoke, and is easily blown about by currents of air. This is the case also with all union jet flames burned at very low pressures. and, practically, a jet of this kind cannot be burned much below 3-10ths or 4.10ths for small sizes, and 5-10ths for large sizes consuming four or five cubic feet per hour. Scholl's "perfecter," as he has called it, has been used extensively in London and other towns for common gas, but it is not suitable for the richer gas used in Scottish towns.

A flame formed by a jet of gas issuing with considerable velocity possesses a certain degree of stiffness, and resists, to some extent, the influences of currents of air. This is particularly necessary in the case of cannel gas, since, whenever the flame is much deflected by air currents, a portion of the carbon arising from the heating of the richer hydrocarbons (*e.g.* olefines, benzole, &c.) passes off unconsumed, and a smoky flame is the result. In practice, it is necessary to sacrifice a certain proportion of the possible illuminating value, in order to give the flame sufficient stiffness to resist currents of air.

Next to the union jet, the "bat's-wing" is that most commonly used for burning gas. It is simply a little tube closed at one end, in which a straight slit is cut, varying in breadth from about 2-10ths to one millimetre. It is made of cast-iron, brass, porcelain, or steatite; the best form being that having a brass body and steatite top. The flame of the bat's-wing is wider and shorter than that of the union jet, and, in order to be equally effective, requires to be burned at lower pressures. It is particularly adapted for large flames burning from 3½ to 5 cubic feet of gas per hour. With rich cannel gas (25 to 30 candles) it gives results at least equal to the union jet, and with gas of 18 to 22 candles, it is decidedly superior.

The following table gives the results of tests of a series of steatite bat's-wing burners manufactured in Germany. Gas, 26 candles:

No. of Burner.	At 0.5 Inch Pressure.			At 1.0 Inch Pressure.			At 1.5 Inch Pressure.		
	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	1.10	4 24	19.27	2.35	9.05	19.25	3.15	11.56	18.35
3	1.45	5.68	19.58	2.65	10.02	18.90	3.55	13.20	18.59
4	1.90	8.76	23.05	3.10	12.71	20.50	4.00	15.41	19.26
5	3.40	16.18	23.80	5.20	24 07	23.14	—	Gas blows.	—
6	4.05	19.09	23.57	—	Gas blows.	—	—	"	—

The considerable loss of light experienced when gas is consumed in bat's wing burners at any but comparatively low pressure, has given rise to many efforts to combine with the jet an apparatus to reduce the pressure of the gas before it issues from the narrow slit. Various burners having obstructions have been constructed, of which Brönner's is one of the best known, It consists of a somewhat pear shaped brass body, with a steatite top, similar to those of which the results are given above, and at the bottom a small piece of steatite in which is an oblong slot. There are, for cannel gas, six sizes of bodies, the sizes depending upon the area of the slots, and five sizes of tops; and as these screw into one another, there are 30 possible combinations. In none of these combinations does the pressure of the gas at the point of ignition, exceed 0.5 of an inch with an

initial pressure of 1.5 inch, while in some it is only 0.2, and in some it is so low that the flame smokes and is useless. The rate of combustion is dependent on three conditions—first, the area of the opening at the bottom; secondly, the area of the slit in the burner; and thirdly, the initial pressure of the gas. The range of combinations enables one to select a burner to suit almost any description of gas or any standard of pressure. The accompanying table gives the results of tests at 1 inch and 1.5 inch, with 26 candles. The burners are not adapted for lower pressures than 1 inch.

For common gas (*i.e.*, of 14 to 16 candles) a different series of tops is provided, in which the areas are considerably greater than in those made for cannel gas, and in which the pressure is reduced to from 0.1 to 0.3 of an inch. These burners cannot be used with cannel gas, although with common gas they are exceedingly effective, and are much in use, especially in London:

BRONNER'S BURNERS FOR CANNEL GAS.

At 1.0 Inch Pressure.					At 1.5 Inch Pressure.				
No. of Burner.	No. of Top.	Cubic Feet per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	No. of Burner.	No. of Top.	Cubic Feet per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	2	1.20	5.07	24.13	2	2	1.40	5.25	18.75
2	3	1.40	6.64	23.71	2	3	1.95	7.37	18.90
2	4	..	Smokes	..	2	4	2.30	10.33	22.46
2	5	..	"	..	2	5	2.40	11.24	23.42
2	6	..	"	..	2	6	..	Smokes	..
2½	2	1.40	5.53	19.75	2½	2	1.90	8.30	21 84
2½	3	1.70	8.48	24.94	2½	3	2.30	10.14	22.04
2½	4	2.03	10.33	25.49	2½	4	2.70	12.08	22.37
2½	5	..	Smokes	..	2½	5	2.85	14.29	25.07
2½	6	..	"	..	2½	6	3.00	15.21	25.35
3	2	1.45	6.27	21.62	3	2	2.00	8 48	21.20
3	3	1.90	8.66	22.89	3	3	2.50	11.34	23.63
3	4	2.13	11.24	26.39	3	4	2.80	14.84	26.50
3	5	..	Smokes	..	3	5	3.15	17.04	27.20
3	6	..	"	..	3	6	3.25	18.07	27.80
3½	2	1.50	5.81	19.36	3½	2	2.12	8.85	20.87
3½	3	1.95	8.30	21.28	3½	3	2.55	12.63	24.76
3½	4	2.55	12.08	23.68	3½	4	3.00	14.47	26.12
3½	5	2.80	14.38	25.68	3½	5	3.50	18.07	25.81
3½	6	3.00	15.38	25.97	3½	6	3.60	19.45	27.01
4	2	1.60	6.36	19.87	4	2	2.30	9.77	21.24
4	3	2.10	10.69	25.45	4	3	2.90	13.83	23.84
4	4	2.65	13.37	25.23	4	4	3.30	17.06	25.85
4	5	3.45	17.61	25.52	4	5	4.10	21.57	26.30
4	6	3.55	18.07	25.45	4	6	4.20	22.40	26.66
5	2	1.77	7.38	20.85	5	2	2.60	9.68	18.81
5	3	2.30	11.90	25.87	5	3	3.30	13.64	20.67
5	4	3.30	15.40	23.33	5	4	4.00	19.91	24.14
5	5	4.10	20.74	25.29	5	5	5.00	25.36	25.36
5	6	4.30	22.68	26.37	5	6	5.30	27.66	26.10

This table shows that it is easy, with properly adjusted bat's-wing burners, to obtain, with a consumption of from 3 to 5 cubic feet per hour, at least the full effect of illumination exhibited in the standard mode of testing already referred to; and that, even with a consumption of 2 cubic feet, a very favorable result may

be obtained. In no case is the loss of light with bat's-wing burners so great as with badly arranged union jets.

Many other descriptions of improved bat'-wings have been constructed, some of which I have tested. The "Clegg" bat's-wing, manufactured by Sugg, has a steatite top, and a conical brass body closed at the bottom, and with a slit cut in it with a fine saw. The respective sizes of the slits above and below determine the consumption of gas and the pressure at the point of ignition. In Silber's bat's-wing made by the Silber Light Company, one burner is placed above another, both being of steatite, the slit of the lower one being much smaller than that of the upper, and connected by a vase of brass. Only the three smallest sizes of these are suitable for rich cannel gas, the larger ones being intended for gas of lower quality. The result obtained with 26-candle gas are given in the following table:

CLEGG AND SILBER BAT'S-WINGS.

Burner.	At 0.5 Inch Pressure.			At 1.5 Inch Pressure.			At 1.0 Inch Pressure.		
	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
Clegg No. 2.	2.00	9.15	22.87	3.40	14.77	21.72	4.45	18.30	20.56
Do. " 3.	2.90	13.00	22.41	4.45	21.11	23.72	5.70	27.04	23.72
Do. " 4.	4.20	20.37	24.25	6.45	31.20	24.19	—	Blows.	—
Do. " 5.	4.80	23.92	24.92	—	Blows.	—	—	"	—
Silber A....	0.95	3.07	16.16	1.50	6.31	21.03	1 90	10.03	26.40
" B....	1.55	7.34	23.68	2.35	12.07	25.68	3.00	15.04	25.07
" C....	2.20	11.24	25.54	3.30	17.27	26.17	4.25	23.12	27.20

Several varieties of regulating bat's-wings have been invented by Sugg, Witthoft, Winsor, and others, the principle of their construction being to check the flow of gas by means of a plug regulated by a screw. At a given pressure in the pipes the burners may be regulated to deliver any desired quantity of gas, and in the experiments on the Winsor and Sugg burners quoted below, they were regulated so as to burn the number of cubic feet per hour corresponding with the numbers marked on the burners. Gas used equal to 26 candles:—

SUGG'S "WINSOR" BAT'S-WING.

No.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	2	9.60	24.00
3	3	15.00	25.00
4	4	19.87	24.84
5	5	25.20	25.20
-	-	—	—

SUGG'S REGULATING BAT'S-WING.

No.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	2	9.20	23.00
3	3	15.34	25.56
4	4	19.90	24.88
5	5	24.75	24.75
6	6	28.74	23.95

If two bat's-wing flames are brought together, especially if the slits be narrow, the gas of low quality, and the pressure somewhat high, the illuminating power of the united flame is greatly in excess of the sum of the two tested separately. Upon this principle is constructed a double-slit bat's-wing, the slits being about one millimèter apart, which is used in Manchester and other towns in England, and which is an excellent burner for gas not exceeding 20-candle power, but gives a somewhat smoky flame with gas of high quality.

The only other bat's-wing that requires

further to be noticed, is the patent regulating bat's-wing used in the United States of America, where it was introduced in 1871, and which is practically the only flat-flame burner capable of burning advantageously the "air gas," made by saturating air with the vapor of petroleum spirit. It consists of a very much elongated iron bat's-wing, with exceedingly narrow slit, surrounded by a brass tube at the distance of about 2 millimèters. Into the space between the two, gas is admitted by a wide orifice (the amount being regulated by a screw), and this gas ascends entirely without pressure, while the force of the gas issuing from the narrow slit spreads it out into a fine soft flame. This burner gives excellent results with gas of all qualities, but its shape is not adapted to the gas-fittings in use in this country, and it has not been used here except for air gas made for private houses.

Argand burners are exclusively used in the photometric testing of common gas, and they are also employed rather extensively for lighting shops and public buildings, but to a limited extent for private houses. They give a higher photometric effect with common gas than any flat-flame burner known; and even with cannel gas, the best descriptions, especially those of Sugg and Silber, give results which approach very near to those obtained when the gas is tested at a comparatively low pressure by large-sized fish-tail or bat's-wing burners.

The original form of Argand was a brass double cylinder with, above, an iron ring perforated with small holes, and below, a "crutch" or forked tube, by which the gas was introduced at opposite sides. A wide and short glass chimney was used, but this was afterwards modified in a variety of ways with a view to making the current of air impinge more directly upon the flame, and so increase the intensity of combustion. The holes being small, the gas escaped at a comparatively high pressure; and the character of the flame, both as to volume, shape, and luminosity, depended partly upon the initial velocity with which the gas escaped from the burner, and partly upon the shape and dimensions of the funnel. The enlargement of the holes, enabling the gas to escape at a moderate pressure, was proposed by the late Dr. Letheby, who was associated with Mr. Sugg, by whom a great many improvements in Argand burners have been introduced. The Letheby burner raised the apparent quality of London gas from 12 to 14 candles, and a further increase of two candles was obtained by Sugg's "London" Argand, now generally accepted as the standard burner for testing gas made from common coal. In this burner the principle is recognized of permitting the gas to escape practically without pressure, the shape and volume of the flame being determined by the narrow funnel and a "cone" of thin metal, which serves to throw the current of air into close contact with the outside of the flame. The upper portion of the burner is of steatite, and, instead of the ordinary "crutch" below, the gas is introduced by three very narrow tubes. A number of sizes of this burner are made, of which details are given below; but the following are the various dimensions of the standard burner used in photometry: Diameter of steatite top, external, 0.84 inch; internal, 0.47 inch; number of holes, 24; diameter of holes, .04 inch; chimney, $6 \times 1\frac{3}{4}$ inches for gas of 14 candles, and 6×2 for gas of 16 candles. The narrow funnel and the cone restrict the quantity of air to very little more than is required to burn the gas, thus avoiding the diminution of light which results from a too rapid combustion of the gas, and the cooling effect of a large volume of air. The pressure of the gas inside the steatite top is considerably less than 0.1 of an inch, and that required to pass five feet per hour through the complete burner is about 0.2 of an inch.

In the burner introduced by Mr. A. M. Silber, the steatite top with wide holes (about one millimèter, or .04 inch) is also adopted; but the body of the burner is considerably elongated, and the so-called "cone" is long and cylindrical, with a curved top. A very essential feature in the Silber Argand is an air-tube introduced into the center of the jet, which is said to carry a portion of the air to the upper part of the flame, and which certainly has a remarkable effect in steadying it. The chimney is $8 \times 1\frac{3}{4}$ inches, and, in consequence of the form of the "cone," is kept so cool at the bottom

that it may be handled without difficulty while the flame is burning. Chimneys of 10 inches high are also used, but, while the consumption of gas is thereby increased, the illuminating power per cubic foot of gas remains almost quite constant. Mr. Silber has discovered the remarkable fact that a globe or vase placed below his Argand increases the illuminating power considerably, and I have had an opportunity of verifying his statement, both as to common and cannel gas, the increase with the former being about a candle, and with the latter about 1½ candles. The effect of placing a vase below an ordinary union jet was also tried, but no increase of light was obtained, while the flame showed a distinct tendency to "blow." The flame of the Argand should have its illuminating power increased 6 per cent. by passing the gas through a glass vase (or cylindical metal box, which answers the purpose equally well) is a phenomenon which appears to be at present incapable of explanation.

The following table gives the results of photometric tests of various Argand burners, with cannel gas of 26 candles illuminating power. From 3 to 4 cubic feet of gas per hour were burned in each case, and the result calculated to the usual standard of 5 feet per hour.

	Size of Chimney.	Illuminating Power.
German porcelain Argand with cone (40 small holes).....	8 × 1¼	17.80
Leoni 40-hole burner, "adamas" top, with cone.....	7 × 1⅞	18.18
Sugg-Letheby 15 holes in steatite ring, perforated gallery ..	7 × 2	18.86
American regulating Argand, brass, 40 very large holes....	5 × 2	21.03
Sugg's "London" Argand, 24 holes, with cone & regulator.	7 × 1¼	22.40
Silber 40-hole burner, steatite top, cone, and centre tube...	8 × 1¼	22 54
Silber 32-hole burner, steatite top, cone, and centre tube...	"	23.08
Silber 24-hole burner, steatite top, cone, and centre tube...	"	25.04
Silber 24-hole burner, with glass vase below.....	"	25.61

The following tests were made with various Argands in order to test the effect produced by the cone and center tube of the Silber burner.

	Pressure at Inlet of Burner Inch.	Gas per Hour Cubic Feet.	Illuminating Power.	Illuminating Power for Five Cubic Ft.
Sugg's "London" Argand, 24 holes.	0.19	3.30	15.00	22.73
Sugg's do., without cone.....	—	2.60	11.80	22.70
Sugg's do., older pattern, 36 holes.	0 17	4.00	16.75	20.94
Sugg's do., without cone.....	—	4.00	17.00	21.25
Silber's 24-hole burner, complete....	0.05	4.00	19.20	24.00
Silber's do., without cone, but with air-tube..... ..	—	4.15	19.00	22.89
Silber's do., without air-tube, but with cone.....	—	3.80	17.20	22.63
Silber's do ,without cone or air-tube..	—	3.40	13.10	19 26

These tests show that the cone, by increasing the draught, enables a larger quantity of gas to be burned, an effect which could be obtained equally well by increasing the height of the chimney; and the air-tube of the Silber burner also produces a similar effect, increasing at the same time the heat and the illuminating power of the flame, and its stability. Indeed, the Silber burner without cone and center tube, and especially when the latter is removed, gives so unsteady a flame that it is practically useless, while in its complete condition it gives the steadiest flame of any Argand yet constructed.

A series of experiments were made in order to ascertain the relative dimensions of the inlet and the outlet of various burners. The upper steatite portion of each burner was removed and fitted up in a little bit of apparatus extemporized for the purpose, so that gas could be passed through the holes, while the bottom portions were simply screwed on in the usual manner, and the gas allowed to escape without lighting it. In all the trials the pressure of the gas was maintained steadily at 0.2 of an inch of water. The numbers represent cubic feet of gas per hour:

	Bottom.	Top.	Complete Burner.
Sugg-Letheby 15-hole burner	16.7	28.7	14.6
Sugg 24-hole standard "London" Argand	4.9	28.8	4.5
Sugg 36-hold (older pattern)	6.1	29.1	6.0
Silber 24-hole	17.7	29.5	17.0
Silber 40-hole	19.1	28.8	18.7

These results show that the pressure of the gas is checked much more efficiently at the bottom of the burner by Sugg's arrangement than by that of Silber, and, in fact, the latter has usually attached to it a small regulator, adjustable by a screw, without which, and when regulated only by a stopcock, a disagreeable hissing noise is produced by the passage of the gas through the almost closed stopcock, unless the latter is far removed from the burner.

The "Bec a Bengel," or Bengel Argand burner, used for gas-testing in Paris, has a porcelain top with 30 rather small holes, a brass cone, and at the bottom what is called a "panier," constructed of porcelain, and pierced with numerous small holes for the admission of air. The chimney is $8 \times 1\frac{3}{4}$ inches. With 26-candle gas it burned 2.5 cubic feet, and gave a light of 10.8 candles, or, for 5 feet per hour, 21.6 candles.

Sugg has constructed a series of "London" Argands, burning from 3 to 12 cubic feet per hour of common gas, and from $1\frac{1}{2}$ to $7\frac{1}{2}$ cubic feet of cannel gas per hour. These from A to I resemble in every respect the standard "London" burner already described; K has, in addition, a single or rat-tail jet in the center, and that marked double is formed of two concentric Argands. They gave the following results:—

(*See Table on following column.*)

It is right to state that all these burners are constructed to burn common rather than cannel gas. A Silber Argand of 24 holes, with chimney 8 by $1\frac{3}{4}$, was tested at the same time for comparison, and gave, for a consumption of 3.75 cubic feet per hour calculated to 5 feet, an illuminating power of 24.02 candles, a somewhat higher result than was obtained with any of Sugg's series, and proving that Silber's Argand is well adapted for burning cannel gas.

Burner.	Number of Holes.	Chimney.	Height of Flame.	Gas per Hour.	Illuminating Power.	Illuminating Power for Five Cubic Feet.
A	15	$6 \times 1\frac{3}{4}$	$2\frac{1}{4}$	1.85	7.67	20.73
B	18	"	$2\frac{3}{4}$	2.65	11.90	22.45
C	21	$6 \times 1\frac{3}{4}$	3	2.85	12.63	22.16
D	24	$7 \times 1\frac{3}{4}$	3	3.25	13.74	21.14
E	27	"	3	3.40	14.67	21.57
F	30	"	$3\frac{1}{4}$	3.72	15.97	21.48
G	33	$8 \times 1\frac{3}{4}$	$2\frac{3}{4}$	4.50	19.13	21.25
H	36	9×2	4	5.05	21.17	20.96
I	40	"	4	5.30	22.30	21.04
K	42	"	$4\frac{1}{2}$	6.50	28.40	21.84
Double	54—21	$10 \times 2\frac{1}{2}$	6	7.80	36.40	23.33

Experiments were made in order to ascertain the loss of light resulting from the use of globes of different kinds, and of various shapes. The loss is always considerable, and in many cases excessive, and it results partly from the absorption of light from the material of the globe, and partly from the draught caused by the ascension of the heated air in the confined space. As regards material, a piece of clear window-glass, held in front of a gas-flame, diminishes the light to the extent of about 10 per cent.; but in the case of a clear globe it is, in some cases, less, owing to the reflection from the surface furthest from the photometer. Globes frosted or ground all over, technically known as "moons," absorb about 25 per cent. of the light when well shaped, and opal or "cornelian" globes 40 to 50 per cent., according to the thickness and quality of the glass. The following results were obtained with globes of different sizes ground all over, and show the effect of increased draught in diminishing the light:—

	Per Cent.
6-inch globe caused a loss of	25
$7\frac{1}{2}$ " " "	$27\frac{1}{2}$
10 " " "	38

All these globes had the usual-sized openings below—about $1\frac{3}{4}$ inches in diameter. Experiments were made with clear $7\frac{1}{2}$-inch globes, having openings below, varying from $2\frac{3}{8}$ inches to 1 inch in diameter. The source of light was a

Brönner bat's-wing, No. 5 top, No. 4 bottom, burning, under a pressure of 1 inch, 3.35 cubic feet of gas.

	Candles.	Per Cent.
The naked flame gave a light of	16.8	
With clear globe,		
opening below 2⅜ in.,	15.4;	loss 8.3
" " 2¼ in.,	15.2;	9.5
" " 2 in.,	13.6;	19.0
" " 1½ in.,	13.0;	22.6
" " 1 in.,	12.0,	28 6

With the two larger-sized openings, the flame was perfectly steady, with that of two inches there was a slight flickering caused by the draught; this was more marked with the 1½-inch opening, and was excessive with that of one inch, making the flame practically useless as a source of light. It is evident, therefore, that the openings of the globes should be as wide as possible, and not less than 2½ inches. The cornelian globes used in Brönner's system of gas lighting have an aperture of 2⅜ inches diameter, and Sugg has introduced globes of similar material which he calls "Albatrine," but with openings of about 4 inches diameter. These globes are constructed of various sizes, to suit certain burners, both bat's-wing and Argand, and the combinations are known by certain names, such as the Westminster, Viennese, Frankfort, Italienne, Parisienne, &c. Some of these arrangements are fitted with regulators, with the intention of maintaining a constant pressure.

One of the difficulties connected with gas illumination is that the pressure in the mains varies considerably in different parts of a town, and at different hours of the day and night. One result is, that a system of lighting, adapted for a part of a town situated in a low level, will show inferior results in a more elevated situation. A rise of 10 feet gives, roughly, a tenth of an inch of increase of pressure, so that it may easily happen that in the same town or city the pressure in one place may be one inch, while in another it may be 2½ inches. Again, the pressure of the gas, as sent out from the gas-works, is altered from time to time, in accordance with the consumption, and as public works, shops, &c., are suddenly lit up or extinguished at certain hours, private consumers are annoyed, in the one case by the falling off in the amount of light, and in the other by a flaring flame and hissing sound, both of which are very irritating. The cure for these evils is to be found in the use of governors or regulators. Every district of a town, the elevation of which is such as to effect appreciably the pressure of the gas, should have a governor, which may either be self-acting, to maintain a constant pressure throughout the day, or to vary sympathetically with the governor at the gas-works. Many of these have been invented, among which may be mentioned those of Cathels, Peebles, and Foulis. The pressure in the mains should not be reduced below 12 or 14-tenths of an inch, but as even that is too high a pressure for the economical burning of gas, each house should have a regulator in order to reduce the pressure constantly to about 7 or 8-tenths. Some of these regulators are dependent on the action of the gas upon a broad leather disc, attached to which is a ball and socket valve, while others have metal or glass bells floating in mercury, and acting upon a valve of the same kind. Both of these work satisfactorly when properly constructed. Among the best dry regulators are those of Sugg of London, and Peebles of Edinburgh, while the best mercurial governor that I have seen is that of Busch of Oldham. In the case of public works and other buildings consisting of several floors, a regulator should be placed on each floor. Street lamp regulators are of great importance, and great attention has been given to the perfecting of them by various ingenious mechanicians. The kind the largest number of which are in use at the present time resembles the dry house regulator already mentioned, the construction being quite similar. These little instruments are made by a great many gas engineers, among whom Sugg and Peebles may be named. The principle involved in the action of the apparatus will be at once understood by a glance at the sectional drawing I have placed upon the wall. It is a regulator, not of volume, but of pressure, and hence the quantity of gas consumed in any street lamp provided with it depends upon the burner. There is an objection to this regulator, and it is a serious one—the leather diaphragm becomes in time hard and stiff, and ceases to act freely, and, unless it be renewed at intervals,

say 12 or 18 months, the instrument is by no means satisfactory. The next street lamp regulator in point of period of introduction is Giroud's rheometer. This beautiful little instrument, which delivers a constant volume of gas, consists of a short cylinder containing glycerine, in which floats a bell of very thin metal, and formed at the top into a cone, the apex of which passes through an orifice in the cover of the cylinder. In the bell itself there is a small hole, through which the supply of gas to the burner must pass. An increase of pressure causes the bell to rise, and the cone to enter the orifice above, thus reducing the area of the aperture through which the gas has to make its way to the burner. The regulation of the rheometer is very perfect, but it ceases to be effective in some eventualities which occasionly occur.

The most recent street-lamp regulator may be called a dry rheometer. It delivers, like the instrument just noticed, a constant volume of gas, but the bell, or substitute for it, instead of floating in a liquid, is simply supported while in action by the pressure of the gas. A regulator of this kind is first indicated in the book published by Giroud, but I am not aware that he ever actually reduced his idea to practice. Victor Bablon, of Paris, patented in 1875, "an apparatus for regulating the flow of lighting gas," in which the float, if it may be so called, is a disc of thin metal connected to a small hollow spindle. It has been introduced somewhat extensively in France, but is almost unknown here. It works with somewhat greater friction than the "needle governor" of Peebles, which is, to my mind, the perfection of gas regulators. In a little cylinder stands a so-called needle, on the point of which rests a flanged cone of exceedingly thin metal. At one side of the cylinder there is a small tube leading away the gas, and by means of a screw working into the side tube, the instrument can be made to deliver any desired number of cubic feet, which it does with surprising accuracy, provided that the pressure of the gas is not less than eight-tenths of an inch. In trials I have made I have not found the variations of volume at different pressures to exceed one per cent. With such a regulator as this, it would be possible to employ Argand burners for street lamps. These burners, when of the best kind, are exceedingly sensitive to quantity of gas. If you have a Sugg or Silber Argand regulated so as to be near the smoking point, and so giving the highest illuminating value that the gas is capable of yielding, the smallest additional supply of gas will cause the flame to smoke. I have here one of Silber's Argands fitted to a needle governor, and you will have an opportunity of seeing the regularity of the flame under different conditions of pressure. I should have mentioned, before the needle governor, the invention of Flürscheim, patented in this country by Borradaile in 1877, but it resembles Bablon's instrument closely, and differs from it chiefly in details.

One other description of regulator remains to be described—that which may be used in connection with the ordinary burners in our apartments. It must necessarily be small, in fact, it should not be much larger than the burner itself. Sugg has, for a number of years, supplied a regulator of this kind, consisting of a leather diaphragm with ball and socket arrangement, but it is a little uncertain in its action, and, so far as my experience goes, not altogether satisfactory; and, besides, it is too large to be useful except as part of a special system of lighting. Peebles has been endeavoring to reduce his needle governor to similar dimensions, and although he does not claim to have yet produced an altogether perfect instrument, in so far as it requires about .8 of an inch of pressure to put it in action, he has great hopes that he will yet be successful.

In this paper I have attempted to indicate the process recently made in the way of developing the photogenic power of coal gas, and the direction in which further improvements may be looked for. I shall be glad if my remarks have the effect of attracting attention to a subject of such interest and importance.

Before concluding, I wish to make one other remark. Last night, on arriving in London, I saw the Thames Embankment illuminated by electricity, and was very much satisfied with the appearance. Then I went across Waterloo Bridge, and saw those lamps in the Waterloo Road; and I must say I was very much struck

with the magnificent light afforded. That certainly was a step in the right direction, although I am of opinion that we are striving too hard to light up our streets. I do not see any necessity for lighting up the streets to make them equal to daylight. What we want is to see where we are going, to avoid being robbed at night, and so on, but I think it is a great mistake that we should strive to make our streets as light as they are during the day. However, there may be economy in these very large burners, and I think there is, though I have not tested them, and I am not able to speak positively. We are very much obliged to Mr. Sugg for lending us these two lamps to-night.

THE TRANSMISSION OF POWER BY ELECTRICITY.

From "The Engineer."

The various reports that have recently been issued relating to the economy of the electric light, all concur in stating that for street purposes gas is more economical than electricity. The concensus of opinion, on the other hand, is rather startling in its unanimity when the lighting relates to large open spaces, or to that which requires penetrating power and brilliancy. This, however, does not surprise any one who understands the theory of the electric light, and the methods of its production. A few weeks since the daily papers, with excess of zeal, produced for the edification of their readers a series of spurious problems, the solution of any one of which was expected to enable electricity to ring the death knell of gas. Among others, the divisibility of the light had great charms, and ever and anon it was discussed as the brilliant discovery of some patentee. The solution of the problem—if, indeed, there is such a problem to solve—still remains amongst the unknown. The multiple are and its capabilities were as well known to electricians ten years ago as they are to-day. Professor Tyndall testifies to this when he says:—"The principles which regulate the division of the current and the development of its light and heat are perfectly well known. There is no room for a 'discovery,' in the scientific sense of the term, but there is ample room for the exercise of that mechanical ingenuity which has given us the sewing machine and so many other useful inventions."

The production of light by electricity has not wholly engrossed philosophers, and of late a good deal has been said concerning the transmission of power to considerable distances by electricity. It may not be uninteresting to explain some of the facts to such of our readers as are not well versed in electrical science. Electricity, as is well known, in many cases acts as if it were a fluid; hence the old hypotheses of one fluid and two fluids. Just as we can lead water by means of pipes from a higher level to a lower level, so we can conduct electricity from one point to another. These points can no more be taken at random with electricity than with water. They must not only be connected by suitable materials, but must be in peculiar electrical conditions. Modern electricians use the term "potential" to indicate an electrical condition, and so state that electricity passes from a point of higher to one of lower potential just as water descends a hill. Whenever and wherever electricity is generated, there is matter existing in these two states, showing, as is commonly said, attractive or repulsive properties, or, more often still, being designated as simply in a positive or a negative condition. We are inclined to the opinion that the phenomena seen are always similarly produced, and that the negative phenomena are due to a differential action; in other words, that electrical repulsion does not exist, but everything is due to attraction. This is heretical, but if we err we do so in good company. One of the fundamental laws of electricity is, that sooner or later the matter thus differently constituted regains its normal condition. We can facilitate or retard this return by the interposition of different materials between the two

points. Thus, if we connect the points by means of a metal wire, the normal state will soon be reached; but, on the other hand, if we use a rod of vulcanite or glass, the return to the normal condition may, under favorable circumstances, be indefinitely retarded. We term metals conductors, because of this property, and such substances as wax, vulcanite, or glass, non-conductors or insulators. The best conductor is silver, but pure copper is almost as good. Then come in the list respectively gold, cadmium, zinc, tin, iron, lead and platinum. Considering these metals, if the conductivity of silver at 0° Cent. or 32° Fah. be represented by 100, that of platinum is nearly represented by 8; increasing the temperature of the metal, however, decreases its conducting power; and at 100° Cent. or 212° Fah. silver and platinum conduct in the proportion of 71 to 6.6. It would not be difficult to explain the cause of the decreased conducting power at the higher temperature; but this is out of our way at present. The fact, however, has to be considered by the electrician when engaged with dynamic electricity, as in the case of the electric light, inasmuch as one of the problems before him is the conduction of a current to a greater or less distance. He requires his current to act at a certain point, and does not wish to lose any more of his power than he can help before he brings it to that point. No substance is a perfect conductor, and no substance is a perfect insulator. Silver presents some resistance to the passage of the current, so does copper and, to a far greater extent, do platinum and iridium. A long wire has a greater resistance than a short wire of the same area, a wire of small section than one of a large. Now the heating effect of a current depends a good deal upon the resistance it encounters. Taking time as constant, the equation $H=C^2R$ gives the heat effect in terms of current and resistance. Another law states that the resistance of a wire varies directly as it length, and inversely as the area of its section, or as the square of its radius. The current, according to Ohm's law, is expressed by the formula $C=\frac{E}{R}$, where E is the electro-motive force of the motor, and R the resistance of the circuit. From such considerations as these it will be seen that calculations can be made as to the size of conductor of any given material required to carry a given current. Dr. Siemens, in his well-known Glasgow address, stated that a 2in. copper wire would convey 1000-horse power from a waterfall to a distance of thirty miles. Mr. T. B. Sprague, one of our more careful and conscientious electricians, in commenting upon this, concludes that, for the above purpose, it would be necessary to provide (1) dynamic converters capable of developing 1000-horse power; (2) 1900 tons of copper rod properly insulated; (3) a dynamo-electric machine capable of generating a current at starting 2000-horse power; and (4) means of cooling that machine, and dissipating rapidly a constant energy equal to 758-horse power. Dr. Siemens admits that, if Mr. Sprague's data are correct, his 2in. wire would have to be increased to 3in., which he maintains would be ample. The cost of such copper wires as these would be enormous, and militate strongly against the general introduction of the electric light. It has been suggested that iron, as being so much cheaper, would obviate this difficulty; but as we have indicated, iron being comparatively a bad conductor, its sectional area must be larger than that of copper, and thus in the long run the cost of conductors would not be greatly diminished by its use. The problem being left in this state in England, Professors Thomson and Houston in America took it up, and their conclusion is rather startling to those who are discussing the problem here. They say: "It is possible, should it be deemed desirable, to convey the total power of Niagara a distance of 500 miles or more by a copper cable not exceeding $\frac{1}{2}$ in. in thickness. This, however, is an extreme case, and the exigencies of practical working would not require such restrictions as to size." We shall now endeavor briefly to indicate the *modus operandi* pursued by these gentlemen, so that readers will be then in possession of the general details of this particular problem.

Suppose two machines connected by a cable of, say, one mile in length, one of these machines being used for the production of a current, the other for the conversion of the current into power.

The two terminals not connected are put to earth. "Let us suppose that the E M F of the current is unity. Since, by the revolution of one machine, a counter electro-motive force is produced in the other, the electro-motive force of the current that flows is manifestly the difference between the two." The resistance of the two machines being made equal to 1, and the connections to .01, the current will be, according to Ohm's law, equal to $1 \div .01 = .99$. If to this system be added an additional machine and a second converter, also another mile of wire, the current will be $2 \div 2.02$, and so on, the addition of 1,000 miles of wire requiring the addition of 1,000 producing and 1,000 converting machines. The expense of this would be very great, and the statement is made merely to show that the problem taken in this light is not unsolvable. In such an extreme case a difficulty would arise in obtaining the requisite insulation, as the electro-motive force would be very great. The authors calculate that "for the consumption of 1,000,000 horse power a cable of about 3 in. in diameter would suffice under the conditions stated." In the results given the authors do not seem to have considered the heating effects upon the wire of small section for short distances, which would, we think, materially affect their calculations; and, again, it must be remembered that to convey the electricity in this way is one thing, but to use the current for lighting purposes is another. The lamps must be arranged either in series or in multiple arc, and while in the latter case the resistance could be kept as low as required, other details would come in so as to require extensive modification of these calculations. The problem then remains—how to convey large currents, obtained from natural sources of power, economically to long distances?

NEW AND SIMPLE METHOD OF RIVER TRAINING.

By A. GEPPERT.

From "Wochenschrift des Oest. Ingenieur-und Architekten Vereines," published by Institution of Civil Engineers.

This mode of river training may be briefly described as consisting in the construction of a partition of sheet piling across the land submerged during floods, and so placed as to connect the two points of the river between which the course is to be straightened. On either side of the sheet piling and along the entire length, a wide trench is excavated, the bottom of which corresponds to the level of ordinary water line. The greater portion of the flood water is thus made to flow off through this artificial channel, and in doing so tends to scour a new river bed in it.

The method has recently been applied in the river Lech, at "The Gechtle," and the author gives an account of the works, and of the results obtained. The pile planking extends over a length of 1,867 feet, and intersects a large island which obstructs the direct course of the river. It is 4 feet high from the bottom of the ditch, and is formed of piles 15 feet 6 inches long, driven at intervals of 6 feet 3 inches, and planked on one side. The piles have a diameter of from $8\frac{1}{4}$ to $9\frac{1}{2}$ inches, and are shod in the ordinary way. The deal planks are $2\frac{1}{16}$ inches thick. The trench was made 6 feet 3 inches wide on each side of the line of piling, the soil of the excavation being simply thrown on the banks. The total cost of the work came to 9*s.* 6*d.* per lineal yard.

During the summer of 1877 various floods occurred in the Lech, which, although continuous, did not attain to any height. The last one alone reached the top of the piling and produced notable changes in the bed of the river, without, however, damaging in any way the planking. In a plan the author shows the course of the river both before and after the flood. It gives a fair idea of the effect which the work has had on the course of the river. After the water subsided it was found that the river had taken a new bed all along the first third of the pile planking. Then, encountering higher ground in front, it had deviated to the left and had fallen back into the former bed of its left channel. On

the other side of the planking, the right branch had also slightly altered its course, and had been considerably reduced through silting. These first results were deemed so satisfactory, considering the small height to which the floods had risen, that it was decided to widen out the unaffected parts of the ditch before next year's floods. It is thought probable that the river will then scour out its bed along the entire length. As soon as this result will have been achieved, the author will report again on the subject.

THERMODYNAMICS.

By HENRY T. EDDY, C. E., Ph. D., University of Cincinnati.

Written for VAN NOSTRAND'S MAGAZINE.

II.

15. THE LAW OF VARIATION OF INTERNAL ENERGY.—When a unit of a given substance is caused to pass from one state of volume and pressure to another, the total increment of its internal energy depends alone upon its initial and final states, and in no way depends upon the route by which the passage is effected.

By (4), $di=dh-dw$ . . . . . (25)

$$\therefore\ i_2-i_1=\int_1^2 dh-\int_1^2 dw \quad . \quad . \quad . \quad (26)$$

In Fig. 2 let the substance pass from state 1 to state 2 by some route x: then, as shown in art. 10, the area between x and the adiabatics e_1 and e_2

$$=\int_1^2 dh=\int_1^2 de,$$

is the heat imparted; and the area included between the ordinates p_1, p_2, x and the axis of v

$$=\int_1^2 dw=\int_1^2 pdv,$$

is the external work performed. Hence by (26) the increment of internal energy is the difference of these areas, but in this subtraction the common area 123 is canceled, and the remainder

$$i_2-i_1=e_2p_2e_1-p_2e_1p_2,$$

is independent of the route x.

It appears from reasoning exactly like that employed in art. 14 that (26), which may also be written

$$i_2-i_1=\int_1^2 tde-\int_1^2 pdv \quad . \quad . \quad (27)$$

is an exact integral, since its value depends alone upon the states 1 and 2.

16. THE LAW OF VARIATION OF INTERNAL WORK.—When a unit of a given substance passes from one state of volume and pressure to another, the total increment of its internal work depends alone upon the initial and final states, and not upon the route.

By (1) $dm=di-ds$ . . . . (28)

$$\therefore\ m_2-m_1=\int_1^2 di-\int_1^2 ds \quad . \quad . \quad . \quad (29)$$

But di is and exact differential by art. 15; and ds is also an exact differential by art. 6 or art. 8: and, as has been shown in art. 14, the statement that any algebraic expression is an exact differential, and the statement that it depends for its value, upon its initial and final states alone, are synomymous.

17. GENERALITY OF THE SECOND LAW.—The second law has been applied in art. 10 to determine the rate at which heat is imparted to a substance. Rankine, however, seems to apply this law only to determine the rate at which external work will be performed. We wish here to point out its wider scope, and have, with this object in view, so worded the statement of it in art. 4 as to express its generality.

The three ways of expending the heat imparted which are detailed in art. 7 are related to each other in some purely dynamical way, such that the proportion of the whole which is expended in each of these separate effects depends on two

considerations only: viz., the kind of substance to which heat is imparted, and the amount of energy acting. The second law asserts that, for a given kind of substance, these effects depend upon the energy acting and not on the state of the substance. The question here suggested is this: What are we entitled to regard as separate "effects" such as are spoken of in the second law? It seems sufficiently clear, that the effects mentioned in art. 7 can be regarded as separate effects to each of which the law applies, and hence to either of their sums. It may be possible to show that some one of these effects is a composite effect, to each of whose parts the law also applies. For example, the internal work may be written thus:

$$dm=\left(\frac{dm}{dt}\right)_v dt+\left(\frac{dm}{dv}\right)_t dv \quad . \quad . \quad (30)$$

$$dm=\left(\frac{dm}{dt}\right)_p dt+\left(\frac{dm}{dp}\right)_t dp \quad . \quad . \quad (31)$$

$$dm=\left(\frac{dm}{dp}\right)_v dp+\left(\frac{dm}{dv}\right)_p dv \quad . \quad . \quad (32)$$

and it would seem that the second law could be applied to some of the components of the internal energy which are expressed by these equations.

Especially is this the case if we accept the molecular theory of matter, for then certain of these components acquire a mechanical significance.

18. Diagrams of Energy.—The diagrams which we have thus far employed, have graphically represented the state of a unit of any given substance by using its volume and pressure as rectangular co-ordinates; and in such a volume-pressure diagram it appears from elementary considerations, that area represents energy or work. In accordance with this idea is the expression for the increment of external work previously employed, viz., $dw=pdv$ . . . . (10) which at the same time is an elementary area of such a diagram.

Equation (10) integrated around the cycle of the indicator card is the work performed per stroke.

It has also been shown in art. 10, that the energy of the total heat imparted in causing a unit of a substance to pass along the route x from the state 1 to the state 2, is represented by the area included between the curves $e_1 x e_2$, when e_1 and e_2 are adiabatics through 1 and 2 respectively, which extend to infinity.

It is, however, equally possible to represent graphically the state of a substance by using as rectangular co-ordinates any two variables which determine its state. In a valuable paper on this subject, J. Willard Gibbs * treats in detail the entropy-temperature diagram, and the volume-entropy diagram.

In an entropy-temperature diagram, it appears from the equation of art. 11, viz.

$$dh=tde \quad . \quad . \quad . \quad . \quad (18)$$

that here also area represents heat imparted or energy; and upon such a diagram, lines of equal volume and equal pressure (called isometrics and isopiestics) can be drawn from experimental data, just as adiabatics (*i.e.* isentropics) and isothermals are drawn upon the volume-pressure diagram. Indeed, the entropy-temperature diagram is found to be specially suited to assist the mind in grasping the thermodynamic relations flowing from the second law.

19. Carnot's Cycle of Greatest Efficiency.—Let a unit of weight of any given substance pass through a series of states represented on a diagram of energy by a closed figure; then, at the end of this complete cycle, the substance is restored to its initial state as to temperature, internal energy, etc. Hence, the area of the closed cycle, which represents the external work, represents also the heat imparted, minus the heat rejected. Draw an adiabatic through each of the two points 1 and 2 of the cycle whose difference of entropy is greatest. Call the upper part of the cycle from 1 to 2 the route x, and the lower part from 2 to 1 the route y, then x and y together form the closed cycle.

The heat imparted while the substance is carried along an element of the route x lying between a pair of adiabatics which are very near each other, is, by (18), $dh_x = t_x\,de$, and the heat rejected on the route y between the same pair of adiabatics is, $dh_y = t_y\,de$.

$$\therefore \frac{h_y}{h_x}=\frac{\int_1^2 t_y\,de}{\int_1^2 t_x\,de} \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

* Graphical Methods in the Thermodynamics of Fluids. By J. Willard Gibbs, Professor of Mathematical Physics, in Yale College. Trans. Conn. Acad. Vol. II, part 2.

is the ratio of the total heat rejected to the total heat imparted.

The heat rejected is a loss, and in order that this loss may be as small a part of the total heat imparted as possible, it is evident that each element $\int_1^2 t_y\,de$ must be as small as possible, while each element of $\int_1^2 t_x de$ must be as large as possible.

Suppose that the highest available temperature of the source of heat is t_1, and the lowest attainable temperature of the refrigerator which receives the rejected heat is t_2, both being of unlimited capacity; then it is evident that the proportion of lost heat is the least possible when $t_x = t_1$ and $t_y = t_2$; and in this case let $h_x = h_1$ and $h_y = h_2$, $\therefore$ by (33),

$$\frac{h_2}{h_1} = \frac{t_2 \int_1^2 de}{t_1 \int_1^2 de} \quad \therefore \text{ by (20)}, \quad \frac{h_2}{h_1} = \frac{t_2}{t_1} \quad . \quad (34)$$

But in case the loss is least the efficiency is greatest.

$$\therefore \frac{h_1 - h_2}{h_1} = \frac{t_1 - t_2}{t_1} \quad . \quad . \quad . \quad . \quad (35)$$

Equation (35) expresses the greatest efficiency of an engine which uses any working substance between the limiting temperatures t_1 and t_2, provided the working substance returns periodically to its initial state.

The conditions of greatest efficiency are then these: all the heat imparted to the working substance must be received by it along the highest available isothermal; all the heat rejected by it must be lost along the lowest isothermal; the passages between the isothermals must be along adiabatics.

This result is independent of the nature of the working substance, which need not be homogeneous, but may undergo changes of state such as liquefaction, vaporization or dissociation in course of the cycle, provided it returns to its initial state at the end of the cycle. It appears from (35) that the efficiency is increased more by decreasing the temperature t_2 of the refrigerator by 1° than by increasing the temperature t_1 of the source by 1°.

20. Partial Differential Coefficients of Volume, Pressure, Entropy and Temperature.*—Let Fig. 3 be part of a volume pressure diagram in which ad is parallel to the axis of v and ad' is parallel to the axis of p. Let tt and $t't'$ be a pair of isothermals whose difference of temperature δt is small. Also let ee and $e'e'$ be a pair of adiabatics whose difference of entropy δe is small.

The pair of isothermals with the pair of adiabatics together inclose a quadrilateral $abcb'$ which is ultimately a parallelogram.

Complete the figure by drawing the various parallels to the axis of v and p which are seen in Fig. 3.

Then, $ad = \delta v$, is the variation of volume when pressure is constant, corresponding to a variation of temperature δt.

This is expressed in the ordinary notation of calculus thus: $ad = \left(\frac{dv}{dt}\right)_p \delta t$ in which the subscript p is used to signify that p is constant in the differential coefficient.

Now since 1° of temperature is an arbitrary magnitude according to the scale adopted, let us assume it small, and let $\delta t = 1°$, then $ad = \left(\frac{dv}{dt}\right)_p$.

The units of distance and force can also be assumed small, so that the unit of work shall also be small. By these assumptions we shall be able to make $abcb'$ differ as little as we please from being an exact parallelogram. Hence we see that if these adiabatics and isothermals form part of a system drawn in accordance with (21), (22) and (24),

then $\quad \delta t = 1, \quad \delta h = 1, \quad \delta e = 1, \quad . \quad (36)$

and ultimately we have,

$$\left.\begin{aligned} ad &= \left(\frac{dv}{dt}\right)_p, & ad' &= \left(\frac{dp}{de}\right)_v, \\ ah &= -\left(\frac{dp}{de}\right)_t, & ah' &= -\left(\frac{dv}{dt}\right)_e, \\ af &= \left(\frac{dv}{de}\right)_t, & af' &= \left(\frac{dp}{dt}\right)_e, \\ ai &= \left(\frac{dp}{dt}\right)_v, & ai' &= \left(\frac{dv}{de}\right)_p, \end{aligned}\right\} \quad . \quad . \quad . \quad (37)$$

* Theory of Heat. J. Clerk Maxwell, Chap. IX.

Again, the areas of the following parallelograms are equal, because standing on the same bases and included between the same parallels.

$$\left.\begin{aligned} abcb' &= abed = adgh = ad.ah, \\ abcb' &= abji = afki = af.ai, \\ abcb' &= ab'j'i' = af'k'i' = af'.ai', \\ abcb' &= ab'e'd' = ad'g'h' = ad'.ah'. \end{aligned}\right\} \quad . . \quad (38)$$

But by art. 13 we may write (36) thus:

$$abcb' = \delta h = 1 \quad . \quad . \quad . \quad (39)$$

$$\therefore\ ad.ah = af.ai = af'.ai' = ad'.ah' = 1 \quad (40)$$

$\therefore$ by (37)

$$\left(\frac{dv}{dt}\right)_p \left(\frac{dp}{de}\right)_t = -1 \quad . \quad . \quad (41)$$

$$\left(\frac{dv}{de}\right)_t \left(\frac{dp}{dt}\right)_v = +1 \quad . \quad . \quad (42)$$

$$\left(\frac{dp}{dt}\right)_e \left(\frac{dv}{de}\right)_p = +1 \quad . \quad . \quad (43)$$

$$\left(\frac{dp}{de}\right)_v \left(\frac{dv}{dt}\right)_e = -1 \quad . \quad . \quad (44)$$

Divide the following equations, member by member, and reduce:

$$(41) \div (42) \therefore \left(\frac{dv}{dt}\right)_p \left(\frac{dt}{dp}\right)_v \left(\frac{dp}{dv}\right)_t = -1 \quad (45)$$

$$(42) \div (44) \therefore \left(\frac{dv}{de}\right)_t \left(\frac{de}{dt}\right)_v \left(\frac{dt}{dv}\right)_e = -1 \quad . . \quad (46)$$

$$(44) \div (43) \therefore \left(\frac{dp}{de}\right)_v \left(\frac{dv}{dp}\right)_e \left(\frac{de}{dv}\right)_p = -1 \quad . . \quad (47)$$

$$(43) \div (41) \therefore \left(\frac{dp}{dt}\right)_e \left(\frac{dt}{de}\right)_p \left(\frac{de}{dp}\right)_t = -1 \quad . \quad (48)$$

Also multiply (45) and (47) member by member and reduce, [*i.e.*, divide the product of (41) and (44) by the product of (42) and (43)], then clear of fractions;

$$\therefore \left(\frac{dv}{dp}\right)_t \left(\frac{de}{dt}\right)_v = \left(\frac{dv}{dp}\right)_e \left(\frac{de}{dt}\right)_p \quad . \quad . \quad . \quad (49).$$

The four partial differential coefficients of (49) are not found in (41) to (44), and their values in terms of the coefficients in (41) to (44) are expressed by (45) to (48). The equations in this article can be proved equally well by considering Fig. 3 to be an entropy temperature diagram.

21. DIFFERENTIALS OF HEAT.—Since the state of a unit of any given homogeneous substance is completely determined when any two of the variables $v\ p\ t$ are given, it is evident that the variation of its state with respect to any of its properties, such as its increment of heat, or its increment of internal energy, etc., can be expressed in terms of the increments of two only of these variables.

$$\therefore\ dh = \left(\frac{dh}{dt}\right)_v dt + \left(\frac{dh}{dv}\right)_t dv \quad . \quad . \quad . \quad . \quad (50)$$

$$dh = \left(\frac{dh}{dt}\right)_p dt + \left(\frac{dh}{dp}\right)_t dp \quad . \quad . \quad . \quad . \quad (51)$$

$$dh = \left(\frac{dh}{dp}\right)_v dp + \left(\frac{dh}{dv}\right)_p dv \quad . \quad . \quad . \quad . \quad (52)$$

For brevity and clearness of conception:—

Let k_v = specific heat with v constant.
" k_p = specific heat with p constant.
" l_v = latent heat of dilatation per unit of increase of v with t constant.
" l_p = latent heat of dilatation per unit of increase of p with t constant.
" h_p = heat received per unit of increase of v with p constant.
" h_v = heat received per unit of increase of p with v constant.

From these definitions we derive the following equations which are further reduced by help of (18) and (41), (42), (43), (44).

$$\therefore\ k_v = \left(\frac{dh}{dt}\right)_v = t\left(\frac{de}{dt}\right)_v \quad . \quad . \quad . \quad . \quad (53)$$

$$k_p = \left(\frac{dh}{dt}\right)_p = t\left(\frac{de}{dt}\right)_p \quad . \quad . \quad . \quad . \quad (54)$$

$$l_v = \left(\frac{dh}{dv}\right)_t = t\left(\frac{de}{dv}\right)_t = t\left(\frac{dp}{dt}\right)_v \quad . \quad . \quad (55)$$

$$l_p = \left(\frac{dh}{dp}\right)_t = t\left(\frac{de}{dp}\right)_t = -t\left(\frac{dv}{dt}\right)_p \quad . \quad . \quad (56)$$

$$h_v = \left(\frac{dh}{dp}\right)_v = t\left(\frac{de}{dp}\right)_v = -t\left(\frac{dv}{dt}\right)_e \quad . \quad . \quad (57)$$

$$h_p = \left(\frac{dh}{dv}\right) = t\left(\frac{de}{dv}\right)_p = t\left(\frac{dp}{dt}\right)_e \quad . \quad . \quad (58)$$

The subscripts of h and k state which one of the variables is considered constant, but the subscripts of l state which one of them is variable.

Equations (50), (51), (52), may now be written

$$dh = k_v\, dt + l_v\, dv \quad . \quad . \quad . \quad . \quad (59)$$

$$dh = k_p\, dt + l_p\, dp \quad . \quad . \quad . \quad . \quad (60)$$

$$dh = h_v\, dp + h_p\, dv \quad . \quad . \quad . \quad . \quad (61)$$

which may be applied to any change of state of a homogeneous substance equally with (3), (4), (5).

1°. Let the change be isothermal, then $dt=0$.

$\therefore$ by (61), $\left(\frac{dh}{dp}\right)_t = h_v + h_p\left(\frac{dv}{dp}\right)_t$. (62)

or $\left(\frac{dh}{dv}\right)_t = h_p + h_v\left(\frac{dp}{dv}\right)_t$. (63)

2° Let the change be at constant volume,

$\therefore$ by (60), $\left(\frac{dh}{dt}\right)_v = k_p + l_p\left(\frac{dp}{dt}\right)_v$. (64)

or $\left(\frac{dh}{dp}\right)_v = l_p + k_p\left(\frac{dt}{dp}\right)_v$. (65)

3° Let the change be at constant pressure

$\therefore$ by (59), $\left(\frac{dh}{dt}\right)_p = k_v + l_v\left(\frac{dv}{dt}\right)_p$. (66)

or $\left(\frac{dh}{dv}\right)_p = l_v + k_v\left(\frac{dt}{dv}\right)_p$. (67)

Equations (62) to (67) may be written by help of (53) to (58) thus:

$$l_p\, l_v = h_v\, l_v + h_p\, l_p \quad . \quad . \quad . \quad . \quad (68)$$
$$l_p\, l_v = t(k_v - k_p) \quad . \quad . \quad . \quad . \quad (69)$$
$$l_p\, l_v = h_v\, l_v - tk_p \quad . \quad . \quad . \quad . \quad (70)$$
$$l_p\, l_v = h_p\, l_p + tk_v \quad . \quad . \quad . \quad . \quad (71)$$

Equations (62) to (67) may also be written thus:

$$\left(\frac{de}{dp}\right)_t - \left(\frac{de}{dp}\right)_v = \left(\frac{de}{dv}\right)_p \left(\frac{dv}{dp}\right)_t \quad . \quad (72)$$
$$\left(\frac{de}{dv}\right)_t - \left(\frac{de}{dv}\right)_p = \left(\frac{de}{dp}\right)_v \left(\frac{dp}{dv}\right)_t \quad . \quad (73)$$
$$\left(\frac{de}{dt}\right)_v - \left(\frac{de}{dt}\right)_p = \left(\frac{de}{dp}\right)_t \left(\frac{dp}{dt}\right)_v \quad . \quad (74)$$
$$\left(\frac{de}{dp}\right)_v - \left(\frac{de}{dp}\right)_t = \left(\frac{de}{dt}\right)_p \left(\frac{dt}{dp}\right)_v \quad . \quad (75)$$
$$\left(\frac{de}{dt}\right)_p - \left(\frac{de}{dt}\right)_v = \left(\frac{de}{dv}\right)_t \left(\frac{dv}{dt}\right)_p \quad . \quad (76)$$
$$\left(\frac{de}{dv}\right)_p - \left(\frac{de}{dv}\right)_t = \left(\frac{de}{dt}\right)_v \left(\frac{dt}{dv}\right)_p \quad . \quad (77)$$

By equations (53) to (58) we also obtain

$$\frac{k_v}{l_v} = \left(\frac{de}{dp}\right)_v = -\left(\frac{dv}{dt}\right)_e \quad . \quad . \quad . \quad (78)$$
$$\frac{k_p}{l_p} = -\left(\frac{de}{dv}\right)_p = -\left(\frac{dp}{dt}\right)_e \quad . \quad . \quad (79)$$
$$\frac{h_v}{h_p} = -\left(\frac{dv}{dp}\right)_e \quad . \quad . \quad . \quad . \quad . \quad (80)$$
$$\frac{k_v}{h_v} = \left(\frac{dp}{dt}\right)_v \quad . \quad . \quad . \quad . \quad . \quad (81)$$
$$\frac{k_p}{h_p} = \left(\frac{dv}{dt}\right)_p \quad . \quad . \quad . \quad . \quad . \quad (82)$$
$$\frac{l_v}{l_p} = \left(\frac{dp}{dv}\right)_t \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Equations (78), (79), (80), and (83) are equivalent respectively to (46), (45), (47) and (48).

We may also write (49) thus

$$k_v\left(\frac{dv}{dp}\right)_t = k_p\left(\frac{dv}{dp}\right)_e \quad . \quad . \quad . \quad . \quad (84)$$

22. Equations of Condition.—By the help of (59), (60), (61) and (10) we may write (25) in either of the following forms

$$di = k_v\, dt + (l_v - p)dv \quad . \quad . \quad (85)$$
$$di = k_p\, dt + l_p\, dp - p dv \quad . \quad . \quad (86)$$
$$di = h_v\, dp + (h_p - p)dv \quad . \quad . \quad (87)$$

But, $dv = \left(\frac{dv}{dt}\right)_p dt + \left(\frac{dv}{dp}\right)_t dp$ . . (88)

$\therefore$ by (86) $di =$

$$\left\{k_p - p\left(\frac{dv}{dt}\right)_p\right\}dt + \left\{l_p - p\left(\frac{dv}{dp}\right)_t\right\}dp \quad (89)$$

Now (85), (89), and (87) are exact differentials by art. 14; hence the "equation of condition of integrability" applies to each of them; and it may expressed thus:

$$\left(\frac{dk_v}{dv}\right)_t = \frac{d}{dt}\left\{l_v - p\right\}_v \quad . \quad . \quad . \quad . \quad (90)$$

$$\frac{d}{dp}\left\{k_v - p\left(\frac{dv}{dt}\right)_p\right\}_t = \frac{d}{dt}\left\{l_p - p\left(\frac{dv}{dp}\right)_t\right\}_p \quad . \quad . \quad . \quad (91)$$

$$\left(\frac{dh_v}{dv}\right)_p = \frac{d}{dp}\left\{h_p - p\right\}_v \quad . \quad . \quad (92)$$

Effecting the differentiations expressed in these equations and reducing we have, by help of (41) to (44) and (55) (56):

$$\left(\frac{dl_v}{dt}\right)_v - \left(\frac{dk_v}{dv}\right)_t = \left(\frac{dp}{dt}\right)_v = \frac{l_v}{t} \quad . \quad . \quad (93)$$

$$\left(\frac{dl_p}{dt}\right)_p - \left(\frac{dk_p}{dp}\right)_t = -\left(\frac{dv}{dt}\right)_p = \frac{l_p}{t} \quad . \quad (94)$$

$$\left(\frac{dh_v}{dv}\right)_p-\left(\frac{dh_p}{dp}\right)_v=1 \quad . \quad . \quad . \quad . \quad . \quad (95)$$

23. The Integrating Factor t^{-1}.—To show algebraically that (59) and (60) have an integrating factor $=t^{-1}$, suppose that the unknown quantity z is an integrating of these equations, then

$$zdh=zk_v\,dt+zl_v\,dv \quad . \quad . \quad . \quad . \quad (96)$$

$$zdh=zk_p\,dt+zl_p\,dp \quad . \quad . \quad . \quad . \quad (97)$$

are exact differentials which fulfill the condition of integrability, which condition may be expressed thus:

$$\frac{d}{dv}\left(zk_v\right)_t=\frac{d}{dt}\left(zl_v\right)_v \quad . \quad . \quad . \quad . \quad (98)$$

$$\frac{d}{dp}\left(zk_p\right)_t=\frac{d}{dt}\left(zl_p\right)_p \quad . \quad . \quad . \quad (99)$$

Effecting the differentiations we have

$$z\left\{\left(\frac{dl_v}{dt}\right)_v-\left(\frac{dk_v}{dv}\right)_t\right\}=k_v\left(\frac{dz}{dv}\right)_t-l_v\left(\frac{dz}{dt}\right)_v \quad (100)$$

$$z\left\{\left(\frac{dl_p}{dt}\right)_p-\left(\frac{dk_p}{dp}\right)_t\right\}=k_p\left(\frac{dz}{dp}\right)_t-l_p\left(\frac{dz}{dt}\right)_p \quad (101)$$

By help of (93) and (94), (55) and (56) we may write (100) and (101) thus:

$$z\left(\frac{dp}{dt}\right)_v=k_v\left(\frac{dz}{dv}\right)_t-t\left(\frac{dp}{dt}\right)_v\left(\frac{dz}{dt}\right)_v \quad . \quad . \quad . \quad (102)$$

$$-z\left(\frac{dv}{dt}\right)_p=k_p\left(\frac{dz}{dp}\right)_t+t\left(\frac{dv}{dt}\right)_p\left(\frac{dz}{dt}\right)_p \quad . \quad . \quad . \quad (103)$$

It is known from the theory of differential equations that z may have, in general, any one of an infinite number of values. If possible, let one of these values have the form

$$z=f(t) \quad . \quad . \quad . \quad . \quad . \quad (104)$$

then, $\left(\frac{dz}{dv}\right)_t=0$, and $\left(\frac{dz}{dp}\right)_t=0$. (105)

Hence (102) and (103) reduce to the same form, viz.,

$$\frac{dt}{t}=-\frac{dz}{z} \quad \therefore \quad z=t^{-1} \quad . \quad . \quad . \quad . \quad (106)$$

This result shows that the supposition made in (104) is possible, and that the unknown form of function in (104) is that expressed in (106).

24. Perfect Gases.—The experimental law of Gay Lussac for perfect gases is

$$\frac{pv}{t}=\frac{p_0v_0}{t_0}=c \quad . \quad . \quad . \quad (9)$$

in which c is a constant to be determined for each gas. Hence for perfect gases,

$$\text{By (55)} \quad l_v=t\left(\frac{dp}{dt}\right)_v, \quad \therefore \quad l_v=p, \quad (107)$$

$$\text{By (56)} \quad l_p=-t\left(\frac{dv}{dt}\right)_p, \quad \therefore \quad l_p=v, \quad (108)$$

$$\therefore \text{ by (69)}, \quad k_p-k_v=\frac{pv}{t}=\frac{p_0v_0}{t_0}=c, \quad (109)$$

Regnault* has shown, experimentally, that k_p, the specific heat at constant pressure, may be regarded as constant for perfect gases, and hence k_v, the specific heat at constant volume, is also constant for such gases.

We can now show that k_p-k_v the additional heat necessary to cause the expansion of the gas is all expended in external work. Since p is constant,

$$\text{by (10)}, \quad \int_1^2 dw=p\int_1^2 dv=p(v_2-v_1) \quad (110)$$

$$\text{By (9)}, \quad \frac{p(v_2-v_1)}{t_2-t_1}=\frac{p_0v_0}{t_0} \quad . \quad . \quad . \quad (111)$$

When $t_2-t_1=1°$, we have by (109), (110), (111),

$$k_p-k_v=p\,(v_2-v_1) \quad . \quad . \quad (112)$$

hence no work is done against internal forces during the expansion, and k_v is the true specific heat. Hence for perfect gases, $dm=0$.

Known gases are very nearly perfect in states distant from the point of liquefaction, but as they approach that point internal attractions are called into play which assist the condensation and cause the volume to be less than that of a perfect gas at the same pressure and temperature.

25. Isothermals and Adiabatics of Perfect Gases.—From (9) it appears that when t is constant,

$$pv=b, \quad . \quad . \quad . \quad (113)$$

in which b is a constant which, by (9), is known when the temperature of the isothermal is given. Hence the isothermals of perfect gases are rectangular hyperbolas, which is the experimental law of Mariotte.

In order to obtain the adiabatics of perfect gases, let us apply the general equations (59), (60), to this case, which by (107), (108), may also be written thus:

* Relation des expériences. Paris, 1862.

$$dh=k_v\,dt+p\,dv \quad . \quad . \quad (114)$$
$$dh=k_p\,dt-v\,dp \quad . \quad . \quad (115)$$

In order to integrate, we must (arts. 14 and 23) divide by t,

$$\therefore \text{ by (109)} \quad de=k_v\frac{dt}{t}+c\frac{dv}{v} \quad . \quad (116)$$

$$de=k_p\frac{dt}{t}-c\frac{dp}{p} \quad . \quad (117)$$

$$\therefore\ e-e_0=k_v\log_{\cdot e}\frac{t}{t_0}+c\log_{\cdot e}\frac{v}{v_0} \quad (118)$$

$$e-e_0=k_p\log_{\cdot e}\frac{t}{t_0}-c\log_{\cdot e}\frac{p}{p_0} \quad (119)$$

These results are general and express the total entropy imparted during any change of state of a perfect gas from any given initial state denoted by the subscript 0.

Now let, $k_p \div k_v = n$ . . . . . (120)

$\therefore$ by (109), $k_v=\dfrac{c}{n-1}$ . . . . (121)

and $k_p=\dfrac{cn}{n-1}$ . . . . (122)

in which $n=1.41$ nearly, for perfect gases.

We may now write (118), (119) thus

$$e-e_0=k_v\log_{\cdot e}\frac{tv^{n-1}}{t_0v_0^{n-1}} \quad . \quad . \quad . \quad (123)$$

$$e-e_0=k_v\log_{\cdot e}\frac{t^np^{1-n}}{t_0^np_0^{1-n}} \quad . \quad . \quad . \quad (124)$$

We may also eliminate temperature from (118), (119)

$$\therefore\ (e-e_0)(k_p-k_v)$$
$$=c\left(k_p\log_{\cdot e}\frac{v}{v_0}+k_v\log_{\cdot e}\frac{p}{p_0}\right) \quad . \quad (125)$$

$$\therefore\ e-e_0=k_v\log_{\cdot e}\frac{pv^n}{p_0v_0^n} \quad . \quad . \quad . \quad (126)$$

In order to obtain the equations which express adiabatic change, in (118), (119), let

$$e-e_0=0 \quad . \quad . \quad . \quad . \quad (127)$$

$$\therefore\ \log_{\cdot e}\frac{t}{t_0}=(n-1)\log_{\cdot e}\frac{v_0}{v}$$
$$=\frac{n-1}{n}\log_{\cdot e}\frac{p}{p_0} \quad . \quad . \quad . \quad (128)$$

$$\therefore\ \frac{t}{t_0}=\left\{\frac{v_0}{v}\right\}^{n-1}=\left\{\frac{p}{p_0}\right\}^{\frac{n-1}{n}} \quad . \quad . \quad (129)$$

$$\therefore\ \frac{p}{p_0}=-\left\{\frac{v_0}{v}\right\}^n,\ \text{or}\ pv^n=p_0v_0^n=a \quad (130)$$

in which a has different constant values for the different adiabatics. The adiabatics of perfect gases are hyperbolic in form, but they cross the isothermals somewhat as represented in Figs. 1 and 3.

26. Flow of Perfect Gases.—Let the converging curved lines in Fig. 4 represent the outlines of a stream of gas moving from a place of greater to a place of less pressure. It is of no consequence whether the sides of the stream be conceived to be bounded by solid walls, or other portions of the gas. Draw any surface cutting all the stream lines of the stream at right angles, and also a second surface infinitely near the first and parallel to it.

Let f=area of the surface cutting the stream.

" dz=thickness of lamina between the two surfaces.

" dp=difference of pressure on opposite surfaces of lamina.

" v=volume of one cubic unit of gas in the lamina.

" v=velocity of the stream.

" g=acceleration of gravitation.

Then, $f\,dz$=volume of the lamina,

" $-f\,dp$=moving force acting on the lamina,

" $\dfrac{1}{gv}$=mass of one cubic unit of gas in the lamina.

" $\dfrac{f\,dz}{gv}$=constant mass of lamina in any position along the stream.

Now from the elementary principles of dynamics the acceleration of any mass is equal to the moving force divided by the mass moved.

$$\therefore\ \frac{d}{dz}\left(\frac{v^2}{2}\right)=-\frac{gv\,dp}{dz}$$

$$\therefore\ -v\,dp=d\left(\frac{v^2}{2g}\right) \quad . \quad . \quad . \quad . \quad (131)$$

As is well known, (131) expresses the energy of motion imparted to a unit of weight of the gas.

In ordinary cases, the efflux of a gas

occurs with such rapidity that no heat can be communicated to it during the process, in which case the efflux is adiabatic.

To find the velocity of flow in this case from the state 1 to the state 2, it will be most convenient to express vdp of (131) in terms of t by (129)

$$\therefore\ v=v_0\left(\frac{t_0}{t}\right)^{\frac{1}{n-1}} \quad . \quad . \quad . \quad . \quad (132)$$

$$dp=\frac{np_0}{(n-1)\,t_0}\left(\frac{t}{t_0}\right)^{\frac{1}{n-1}}dt \quad . \quad . \quad . \quad (133)$$

$$\therefore\ \frac{v^2}{2g}=\frac{-cn}{n-1}\int_1^2 dt=\frac{cnt_1}{n-1}\left(1-\frac{t_2}{t_1}\right) \quad . \quad (134)$$

From (134) the velocity of efflux v can be expressed by (129) in terms of the initial and final pressures or volumes, as well as in terms the temperatures employed in (134), for by (129)

$$\frac{t_2}{t_1}=\left(\frac{v_2}{v_1}\right)^{1-n}=\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}=r \quad . \quad . \quad . \quad (135)$$

$$\therefore\ \frac{v^2}{2g}=\frac{cnt_1}{n-1}\,(1-r) \quad . \quad . \quad . \quad . \quad (136)$$

in which we can use either value of r given in (135).

The greatest velocity occurs when

$$r=0, \quad \therefore\ v=\left(\frac{2gcnt_1}{n-1}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (137)$$

is the velocity with which a gas flows into a vacuum.

But to find the conditions for the flow of the maximum quantity per second, it is necessary to make $v \div v_2$ a maximum; for this is the weight of gas flowing across a unit area per second.

$$\frac{v}{v_2}=p_1 r^{\frac{1}{n-1}}\left\{\frac{n(1-r)}{c(n-1)t_1}\right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (138)$$

$$\therefore\ r=\frac{2}{n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

is the condition for a maximum.

$$\therefore\ \text{by (136)},\ v=\left(\frac{2gcnt_1}{n+1}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (140)$$

is the velocity of maximum flow.

27. Perfect Gas Flowing into a Vacuum.—Suppose that any gas, as the atmosphere, flows at a constant initial temperature t_1 and pressure p_1 into a receiver in which the temperature is $t_2=nt_1$, in which n has the value before given.

It is evident that the work done by the exterior pressure in forcing a unit of gas into the receiver is $p_1v_1=ct_1$, and the energy already existing in a unit of gas as sensible heat is, by art. 24, $k_v\,t_1$ for k_v is the true specific heat; hence, the total energy gained by the receiver with each unit of gas is the sum of these quantities.

$$\therefore\ \text{by (121)} \quad (c+k_v)t_1=nk_v\,t_1 \quad . \quad (141)$$

is the total energy gained with each unit of gas. This energy gained, if used only in heating the unit of gas which carries it in, will raise that unit to a temperature,

$$t_2=nt_1 \quad . \quad . \quad . \quad . \quad (142)$$

since k_v will raise it 1°. But when the gas already in the receiver has a temperature $t_2=nt_1$, the energy of the gas which enters will be employed in heating itself alone.

When the receiver is empty at the beginning of the flow, the energy carried into it can be expended only in heating the gas itself. The gas consequently rises to the temperature $t_2=nt_1$, and continues at that constant temperature during the influx, provided no heat is lost or gained by processes other than those mentioned. This remarkable result was first obtained, but in a different manner, by Bauschinger.

28. Flow of Imperfect Gases.—No one of the more perfect gases obeys precisely the law of Gay Lussac, when the absolute zero of temperature is that defined by the third law. But each gas may be taken to obey a law expressed by the equation

$$\frac{pv}{t'}=\frac{p_0v_0}{t_0'}=c' \quad . \quad . \quad . \quad (143)$$

in which t' is temperature measured by a thermometer constructed, as in art. 9, of the particular gas under consideration. In that case t' will differ slightly from t, and the difference, $t-t'$, will be different for different gases. By reason of difficulties met with in experimenting on gases at widely different pressures, Thompson has proposed to measure the amount by which any gas diverges from the law of Gay Lussac by causing it to flow through a tube at one point of which is an obstruction, called a porous plug, which causes a slight difference of press-

ure in the tube on either side of it. If the gas is imperfect, there will be a slight change of temperature after passing the plug, as may be seen from the following discussion:

By (56) and (60), $dh=k_p\,dt-t\left(\frac{dv}{dt}\right)dp$ (144)

Now let the difference of pressure on the two sides of the plug be dp, then

by (131), $dh=-vdp$ . . . (145)

for the only energy supplied, when the gas forms a steady stream, is that producing the motion through the plug. Therefore by (144) and (145)

$$k_p\,dt=\left\{t\left(\frac{dv}{dt}\right)_p-v\right\}dp \quad . \quad (146)$$

which vanishes for a perfect gas, but in an imperfect gas

by (143), $k_p\,dt=c'(t-t')\,\frac{dp}{p}$. (147)

In integrating this equation $(t-t')$ is considered constant for any given gas within the range of the experiment,

$$\therefore\quad k_p\,(t_2-t_1)=c'(t-t')\log.\,\frac{p_2}{p_1} \quad . \quad (148)$$

In (148) we can without sensible error measure the difference of temperature by the gas thermometer, *i.e.*,

$$t_2-t_1=t_2'-t_1' \quad . \quad . \quad . \quad . \quad (149)$$

Hence $t-t'$ can be found, since all the other quantities in (148) can be determined experimentally.

Experiments by Joule and Thompson gave a slight decrease in temperature after passing the plug for all the permanent gases except hydrogen, which was heated very slightly. Air was cooled slightly, but carbonic acid gas was cooled much more than oxygen, nitrogen or air. The cooling effect is much diminished at high temperatures, and there is doubtless a particular temperature for each gas at which it is perfect.

29. Sound in a Perfect Gas.—For the sake of simplicity, consider a uniform tube of the gas extending in the direction in which the sound is propagated, and having a cross section of one square unit. Suppose that the tube is cut at right angles by two moving planes which remain at a small constant distance from each other, and that both of these cutting planes move along the tube with the velocity with which sound itself moves. Let the first plane cut the tube at a point where the specific pressure and volume of the gas are the same, as in the still gas which is unaffected by the sound wave. Then the lamina between the planes receives through the front plane the same quantity of gas per second as if moving with the same velocity through still gas. Also, since the lamina, in moving with the same velocity as the sound wave, remains stationary in the same part of the wave its density remains stationary, hence it rejects the same quantity of gas through the second plane that it receives through the first.

Let v = velocity of sound which is also the velocity of the gas at first plane.

" $\mathrm{v}+dv$ = velocity of gas at second plane.

" v = volume of a unit of gas at the first plane.

" $v+dv$ = volume of a unit of gas at the second plane.

" q = quantity of gas, by weight, entering and leaving the lamina per second.

" dp = difference of pressure at first and second planes.

Then; $\mathrm{v}=qv$ . . . . . (150)

is the volume swept through by either plane face of the lamina.

Also, $dv=qdv$ . . . . (151)

is the difference of velocities of the gas at the two faces of the lamina.

And, $-dp=\frac{q}{g}d\mathrm{v}$ . . . (152)

is the force acting on the mass $q\div g$ to cause the acceleration $d\mathrm{v}$.

$$\therefore\ q^2=-g\frac{dp}{dv},\ \therefore\ \mathrm{v}^2=-\left(\frac{dp}{d\,v}\right)_e gv^2 \quad . \quad (153)$$

The propagation of sound is so rapid that the gas undergoes an adiabatic change, and this is expressed by making entropy constant as we have done in (153).

By (130), $\left(\frac{dp}{dv}\right)_e=-\frac{np}{v}$, $\therefore\ \mathrm{v}^2=ngpv$ (154)

$\therefore$ by (109), $\mathrm{v}=\sqrt{cngt}$ . . . (155)

which is the ordinary formula for the velocity of sound.

By (137) the velocity with which a gas enters a vacuum exceeds the velocity of sound in the ratio

$$\sqrt{2} : \sqrt{n-1} = 2\tfrac{1}{2} \text{ nearly.}$$

By (140) the velocity of greatest flow is less than the velocity of sound in the ratio

$$\sqrt{2} : \sqrt{n+1} = \tfrac{9}{10} \text{ nearly.}$$

THE THEORY OF TURBINE WHEELS.

A REVIEW OF PROF. TROWBRIDGE'S RECENT ARTICLE.

By Prof. WM. H. BURR.

Written for Van Nostrand's Magazine.

From the deservedly high position occupied by Professor Trowbridge, his late article on Turbines will be likely to convey permanent impressions of a very erroneous character.

His objection seems to be, that Weisbach, Rankine and others "insist" that, for the best performance of a wheel, the water shall enter without shock; while he holds that an equal efficiency, at least, may be obtained if the relative velocity be not tangential to the buckets at the point of entrance into the wheel. In other words, if resistances be left out of consideration, it is a matter of no importance whether there is or is not shock. Now if the consideration of resistances be omitted, and if "shock," be of no importance one way or the other, then it follows that a formula for curved floats ought to give precisely the same result whether the relative velocity of entrance be tangential in direction or not. In fact the formulæ of Weisbach show this exactly, as do those also of Bresse and Rankine. In order to verify this statement let any one turn to the expressions for the efficiency of a turbine in Art .260 of Dubois' translation of Weisbach's work on water wheels, Eq. 21, Art. 15 of Mahan's translation of Bresse on "Hydraulic Motors" (John Wiley and Son, 1869), and Eq. 6 Art. 174 of Rankine's "Steam Engine and other Prime Movers," 5th edition; in no one of these expressions is there anything found depending upon the angle which the bucket makes with the circumference of the wheel at the point of entrance into it. Now the matter of shock, or no shock, depends upon the value given to that angle and since the expressions for the efficiency do not in any wise depend upon such a value, they manifestly are *not* based upon the "theorem or axiom" that the "water must enter the wheel without shock."

What those authors *say* is that the "water must enter the wheel without shock" in order to get the best results, simply because, if there is shock, eddies, consequent upon the impact of *water in water*, will be produced giving rise, as is well known, to no small loss of energy, so far as useful effect is concerned. This element is of such uncertain value, however, in a turbine that the authors above mentioned made no attempt to allow for it in their expressions for efficiency, but state explicitly that the wheels which they treat are supposed to run under the most favorable conditions possible.

Again in the first part of his article Professor Trowbridge seems to hold that what he considers the erroneous views of the authors above cited, leads them to make what Weisbach designates as the angle β so large, that no work can be done by *impulse*, and would seem to imply that this is a disadvantage. Now Bresse, in Art. 16 of the work already named, advises that this angle be made a little less than 90°, *which will allow of some work being done by impulse.*

Weisbach, however, in art. 254 of Du Bois' translation, advises as high as 100° to 120°, and so far as I am able to determine from the data before me, Mr. Francis has never made the angle less than 90°, but as high as 119° in his "Centre-Vent" Boott wheel, thus effectually preventing all work by impulse.

Prof. Trowbridge's objections, there-

fore, can not in any sense hold against the investigations of the authors named. The faults on which he bases his objections do not exist.

The influence of the indirect effect of shock in loss of energy by induced eddies, in Weisbach's formula for the efficiency may easily be seen by referring to the Art. 260 already cited. His expression for the efficiency is;

$$\eta=1-\frac{\zeta\left\{1+\left(\frac{r\sin\delta}{r_1\sin a}\right)^2\right\}+4\left(\sin\frac{\delta}{2}\right)^2}{\zeta\left\{1+\left(\frac{r\sin\delta}{r_1\sin a}\right)^2\right\}+2\cot a\sin\delta} \quad \ldots\ldots(a)$$

Now the manner in which that equation is established shows that the loss of energy by the shock considered would cause a term of the form, $\zeta_1 f(\beta)$, to enter the numerator of the second term of the second member; ζ_1 being a co-efficient of resistance for the impact of water in water and $f(\beta)$, a term dependent upon the angle β, the angle at which the buckets cut the circumference of the wheel at the point of entrance into it. It is evident that this term would diminish the efficiency, as it should.

Mr. Francis himself virtually points out the existence of this term for his "Centre-Vent" wheel, on page 69 of Van Nostrand's last edition of his "Lowell Hydraulic Experiments," though in that case the impact is, to a considerable extent, probably, due to another cause which will be indicated further on.

It will now be shown that the formula for the efficiency given by Bresse is identical with Prof. Trowbridge's, and that the same holds for Weisbach's when resistances are omitted, which is assumed by Prof. Trowbridge.

The general expression for the efficiency which Prof. Trowbridge arrives at is

$$E=1-\frac{v_r^2\sin\gamma^2}{v_1^2}$$

Let $v_r \sin\gamma$, the absolute velocity of escape from the wheel, be represented by w; then

$$E=1-\frac{w^2}{v_1^2} \quad \ldots\ldots(b)$$

The quantity v_1 is the velocity due to the whole head available in any particular case.

On page 89 of the edition of Mahan's Bresse, already noticed, if v be put for v' and v_1^2 for $(2gH)$, as must be done to bring both formulæ to the same notation, there results this expression for the efficiency, which is identical with Eq. (b):

$$\mu=1-\frac{w^2}{v_1^2} \quad \ldots\ldots(c)$$

In Eq. (a), Weisbach's expression, if resistances be omitted and H represent the total fall, there will result;

$$\eta=1-\frac{4v^2\left(\sin\frac{\delta}{2}\right)^2}{v^2\cot a.\sin\delta}$$

But Weisbach shows that,

$$w^2=4v^2\left(\sin\frac{\delta}{2}\right)^2,$$

and

$$2gH=v_1^2=2v^2\cot a.\sin\delta;$$

hence

$$\eta=1-\frac{w^2}{v_1^2} \quad \ldots\ldots(d)$$

This again is Professor Trowbridge's formula.

It is clear, therefore, that any objection which holds against the formulæ of Weisbach, Bresse, and consequently Rankine, holds with equal force against the formula of Professor Trowbridge. Also, that if there be omitted from Weisbach's expression that which makes it of real practical value, or, in other words, if we make it about as loose an approximation as we can well have (since the efficiency must be less than unity in any case), there results the formula of Professor Trowbridge. Equations (b), (c) and (d) are, however, all correct under the supposition of no resistances.

The expressions for efficiency and best velocity of rotation arrived at by Weisbach will be tested by the results of some of the best experiments on Turbines that probably were ever made.

Reference is made to the experiments of Mr. Francis in the Tremont Turbine and Centre-Vent Wheel at the Boott Cotton Mills, as given in the third edition of his "Lowell Hydraulic Experiments;" also his experiments on a Swain Turbine, published in the *Journal of the Franklin Institute* for April, 1875; and, finally, to the experiments on a Swain Turbine by Hiram F. Mills, C.E., at Lowell, Mass.,

in June 1869, published in the Franklin Institute Journal and republished in 1872 by the Swain Turbine Company in a business pamphlet.

The experiments chosen will be those for a full gate only, as those are the only ones to which the formulæ directly apply.

Some of the angles used I have been obliged to measure from such plates as I have at hand, and it is barely possible that the values taken may be a little in error, but not not enough to make any sensible difference in the results.

The extreme outer radius of the Fourneyron wheel and the interior radius of the inward and downward flow wheels, have been diminished by a little less than one-half the shortest distance between the buckets for obvious reasons.

The notation used is that of Weisbach, and has no connection with that used before.

Weisbach's expression for efficiency, as already explained, is eq. (a); his formula for best velocity of the interior circumference of the Fourneyron and exterior circumference of the others, is found in Art. 251 of Du Bois' translation before mentioned; it is denoted by v_1. Inasmuch as Weisbach's formulæ are to be tested, his value of $\zeta=0.075=\zeta_1$ will be used.

TREMONT WHEEL.

For experiment No. 30, 3d edition, Lowell Hydraulic Experiments, pages 32 and 33:—

$\alpha=28°$ $\quad r=4.1$ ft.
$\beta=90°$ $\quad r_1=3.4$ ft.
$\delta=11°$ $\quad h=12.9$ ft.
$v_1=0.66\sqrt{2gh}=19.2$ ft.
Experiment gives: $\quad v_1=18.05$ ft.
$\eta=1-0.16=0.84$
Experiment gives: $\quad \eta=0.794$

CENTRE-VENT BOOTT WHEEL.

For experiment No. 30, pages 66 and 67, 3d edition, Lowell Hydraulic Experiments—

$\alpha=9°.5$ $\quad r=3.95$ ft.
$\beta=119°.0$ $\quad r_1=4.67$ ft.
$\delta=11°.0$ $\quad h=13.378$ ft.
$v_1=0.716\sqrt{2gh}=21.01$ ft.
Experiment gives: $\quad v_1=19.71$ ft.
$\eta=1-0.0756=0.924$
Experiment gives: $\quad \eta=0.797$

This particular experiment was taken because the results are about the mean of those of seven, giving essentially the same efficiency.

SWAIN TURBINE.

Experiment 134 of those made by Mr. Francis at Lowell, in Feb., 1875, and published in the *Journal of the Franklin Institute* for April, 1875.

The value of δ is taken to be a half of the sum of its values at the highest and lowest points of exit from the wheel. It is not at all probable that this is *exactly* right, but it is about as near as can be obtained, and it is evident that not much of an error is involved.

All molecules of water, also, have approximately the same length of path in the wheel, hence the value of r is taken as if the wheel were wholly an inward flow one

$\alpha=14°$ $\quad r=22.5$ inches.
$\beta=95°$ $\quad r_1=36.0$ inches.
$\delta=\frac{18+7}{2}=12°.5$ $\quad h=12.41$ feet.
$v_1=0.70\sqrt{2gh}=19.8$ feet.
Experiment gives: $\quad v_1=21.48$ feet
$\eta=1-0.079=0.921$
Experiment gives: $\quad \eta=0.834$

SWAIN TURBINE.

Experiment, No. 11 of the series, made by Hiram F. Mills, C.E., at Lowell, Mass., June, 1869.

Observations made on the data of the previous example, apply here also.

$\alpha=22°$ $\quad r=12.2$ inches.
$\beta=104°$ $\quad r_1=21.0$ inches.
$\delta=\frac{20+7}{2}=13°.5$ $\quad h=14.255$ feet.
$v_1=0.723\sqrt{2gh}=21.9$ feet.
Experiment gives: $\quad v_1=21.8$ feet.
$\eta=1-0.113=0.887$
Experiment gives: $\quad \eta=0.822$.

Weisbach's formula gives the efficiency of the wheel simply as a user of water, implicitly considering, therefore, the wheel friction as useful work. If Prof. Trowbridge's figures, therefore, be taken, there should be added to each of the above experimental efficiencies the frac

tion 0.03. Then taking them in the order of the examples they would stand as follows:

$$\eta = 0.824$$
$$\eta = 0.827$$
$$\eta = 0.864$$
$$\eta = 0.852$$

A critical comparison may now be made between the results of calculation and experiment. Mr. Francis' table for the Tremont Turbine shows that, with an essentially full gate, v_1 in the six experiments for which the efficiency was 0.79 or over, varied from 17.3 feet to 18.40 feet, while the formula gave $v_1 = 19.2$. With this last value of v_1 the Tremont Turbine gave a corrected efficiency of over 0.81.

The corrected experimental efficiency above is 0.824; the formula gives 0.84; this disagreement is certainly small.

The analytical results, then, for the Tremont Turbine surely are to be classed as excellent, and formulæ which can be depended upon to give as good results are worthy of the confidence of an engineer.

The analytical and experimental results for the second example, the Center-Vent Wheel, do not agree as closely, and, indeed, an examination of the wheel shows why they can not. The distance between the wheel and the guides is more than two-thirds the shortest distance between any two consecutive guides, and the angle α is taken so small (9.5° only) that the entering stream is directed nearly tangentially to the wheel. There must result, therefore, some impact of water in water in the annular space between the guides and buckets, consequently, considerable loss of energy by eddies, etc., will follow. The angle β was also so chosen, as Mr. Francis himself shows (page 69), that there is appreciable impact of water in water at the section of entrance into the wheel.

This shows why there ought not to be any impact in entering the wheel, and it is the very thing that Wiesbach, Bresse, and Rankine guard against.

The investigations of those authors show that loss of energy was to have been anticipated from such injudicious choice of the angles α and β. It is to be noticed that much larger values of α are found in the Swain Turbines taken, and that considerably larger efficiencies are obtained.

It has already been shown how the formula for the efficiency should be changed to meet this case, and how it (the efficiency) would be decreased by the change. The analytical value of v_1 is certainly as near the experimental as could be expected under these circumstances. It will be shown further on how a term can be introduced in the value of v_1 so as to make it apply to this case, and that the effect of such a term would be to decrease the velocity v_1. The discrepancies, then, in this case should really strengthen one's confidence in the formula; in other words they are confirmatory. This is so because the formulæ do not apply to all the circumstances of the present case, whereas if the proper terms were introduced in accordance with the general theory on which they are based the resulting changes would be in the proper direction.

It will also be shown that the formulæ of Wiesbach indicate a high pressure at the point of departure from the guides, and the result is confirmed by the experiments. There would be considerable loss under this head but for the admirable arrangements for the prevention of leakage.

The results in the third example are very good. The disagreement in the efficiencies is between 0.05 and 0.06, and in the values of v_1 about the same portion of $\sqrt{2gh}$. Experiment 137 shows that with v_1 equal to $0.70\sqrt{2gh}$, nearly, the resulting efficiency was about 0,85, when corrected for wheel friction.

Bearing in mind, also, that exact values for r and δ could not be obtained, it is clear that the operations of the formulæ are most creditable.

The analytical results in the last case agree so nearly with the experimental that they are almost suspicious, nevertheless they are honest calculations and show how thoroughly reliable the formulæ are.

Some other considerations, still more confirmatory, if possible, of Wiesbach's formulæ remain to be noticed.

He shows that if β be greater than 2α, there will be unbalanced outward pressure at the point of exit from guide curves. In the Centre-Vent Boott Wheel the pressure head at that point may be calcu-

lated from the amount of water discharged and the total area of exit guide orifices, and there is found an unbalanced head of water equal to about 7.7 feet. This is a confirmatory result, because in this case β is several times greater than 2α.

In the Tremont Turbine there will be found an unbalanced pressure head at the same point of about 6 feet. This was to be anticipated, since $\beta > 3\alpha$.

The formulae indicate, therefore, just what the experiments show, *i. e.*, if small values of α are used, then unusual precautions must be taken to prevent leakage; this certainly was effectually done in the Lowell wheels, including the Swain.

Weisbach and Bresse seem to lay unnecessary stress on the point of taking α and β so that there will be no unbalanced outward pressure at the exit from the guides, for the following reasons:

Equation (a) shows, since the cotangent of a small angle is very large and increases very rapidly as the angle decreases, that a small value of α will give a considerable increase of efficiency over that which will be obtained by a little larger value. It follows from the formulae, therefore, that it is best to take precautions against leakage and make α small, as was done in the Swain Turbines at Lowell.

The general theory of hydraulic resistances, however, show that a minimum limit may be reached below which it is not best to go. For with a very small angle, α, the stream issuing from any guide orifice becomes very thin in comparison with the width of the annular space occupied by the regulating gate, and its direction becomes nearly tangential to the wheel; disturbances caused by impact of water in water result, and there is an induced loss of energy by eddies, &c. Farther if the angle β is not properly chosen the same kind of impact and loss of energy will occur in the wheel.

Figs. 1 and 2 above illustrate this last matter. Fig. 1 belongs to experiment 30 on the Boott Centre-Vent Weeel already treated; ad represents $v_1 = 19.7$ feet; ac, the velocity of exit from the guides, calculated from Mr. Francis' data and found equal to 19.1 feet; and ab the consequent velocity relative to the wheel. The angle bad is found, thus, to be 105°; this should have been the angle β for the actual value of v_1. The value taken, however, was 119° and loss of energy by impact of water in water, induced by shock, resulted as Mr. Francis shows.

Fig. 1.

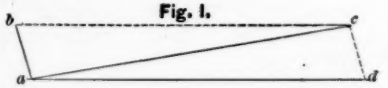


Fig. 2.

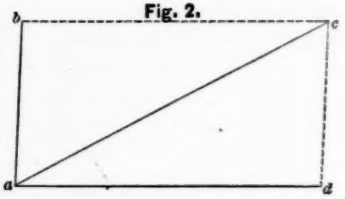


Fig. 2 belongs to the experiment 30 in the Tremont Turbine; $ad = v_1 = 18.05$ feet; ac = exit velocity from guides = 21.1 feet; and ab is the velocity relative to the wheel. The angle bad is found to be 87°, while β was taken at 90°. There can be little or no impact in this case, as the analytical results show.

From the manner of establishing Weisbach's value for v_1, the best velocity of rotation, it will be seen by examination that the case of a partially closed gate may be allowed for by placing for ζ the expression $\left\{\zeta + \zeta_1 . f\left(\frac{1}{h}\right)\right\}$ in which $f\left(\frac{1}{h}\right)$ is a function of the reciprocal of the height of the regulating gate, and ζ_1 an empirical quantity dependent upon the impact of water in water.

Since this will increase the denominator of the expression for v_1, it follows that the formula would indicate a decrease of the best velocity for a partially closed gate. An examination of the experimental results is confirmatory. Let the experimental results of Mr. Francis in the seventy-two inch Westerly Swain Turbine already used be referred to. Then taking from two to four of the best results (whose efficiences do not differ essentially from each other and from the highest) for each of several different heights of regulating gates; the following quantities are then deduced:

Experiments 133, 134 and 135:
Height of gate, 13.08 inches.

Mean $v_1 = 0.757\sqrt{2gH}$.

Experiments 117 and 118:
Height of gate, 11.00 inches.

Mean $v_1 = 0.741\ \sqrt{2gH}$.

Experiments 102, 103 and 104:
Height of gate, 9.00 inches.

Mean $v_1 = 0.705\sqrt{2gH}$.

Experiments 77, 78, 79 and 80:
Height of gate, 7.00 inches.

Mean $v_1 = 0.699\sqrt{2gH}$.

Experiments 50, 51 and 55.
Height of gate, 5.00 inches.

Mean $v_1 = 0.688\sqrt{2gH}$.

Experiments 33, 34 and 55:
Height of gate, 3.00 inches.

Mean $v_1 = 0.659\sqrt{2gH}$.

The variation in these results is as nearly uniform as could be expected under the circumstances, since only from two to four results in each case could be found with efficiencies near enough the maximum. They show, however, just what the formula would indicate, *i.e.*, a decrease in v_1, for a decrease in height of the regulating gate.

The other series of experiments will be found on examination to show the same thing, and it is not necessary to reproduce them here.

The formula for the efficiency would show a decrease in it, also, as has before been indicated.

There is only one thing more to be touched upon, which, indeed, might have been alluded to before. If the four wheels examined had been run with the best velocities indicated by the formula of Weisbach, then the experimental results show that in no case would the efficiency have fallen below the maximum actually obtained, by more than the decimal 0.015. Considering the complicated action of the water in a Turbine this is certainly a most excellent result.

Summing up then, it is seen that for the effect of a partial gate on efficiency and best velocity; for the effect of the relative values of a and β on the unbalanced pressure at point of exit from guides, and impact in the same vicinity, the formulæ indicate just the kind of variation which the experiments show.

The qualitative characteristics of the formulæ are correct; the empirical quantities which enter them only need to be determined by such admirable experiments as those of Mr. Francis, to give the quantitive results their exact values.

The formulae for efficiency and best velocity, as they stand, give most excellent results for full gate and proper design.

In the light of these facts it can hardly be said that mathematical investigation has done little or nothing to aid the engineer in designing the turbine, or that the formulae of Weisbach are inapplicable to modern Turbines.

Before closing it may be well to state that Professor Trowbridge makes a wrong use of the co-efficient of resistance

$$\left\{\left(\frac{1}{0.97}\right)^2 - 1\right\},$$

for he multiplies it by the square of a velocity which does not exist at the point of exit from the guides. His v_1, as he uses it, is the velocity due to the total fall, and that is a very different quantity from either the velocity in the guides or that in the wheel, and the latter are the ones he should use.

ERRATUM—THE MISSISSIPPI AS A SILT BEARER.—There was an unfortunate transposition of the text of the above article in our last issue.

To be read as was intended, the reader should pass from the paragraph ending on 3d line from foot of 2d column, page 222 to the paragraph beginning on the 12th line 1st column, page 226.

The omitted text should be inserted immediately after the paragraph in the 2d column, page 230, which ends with "foregoing table."

REPORTS OF ENGINEERING SOCIETIES.

ENGINEERS' CLUB OF PHILADELPHIA.—The first issue of the Proceedings has just come to hand. There is much promise of valuable additions to engineering literature by this active organization.

The following is the list of papers in the current number:

I. Oil Lands of Pennsylvania. Charles A. Ashburner.

II. House and Street Drainage of Philadelphia. By Rudolph Hering.

III. Bearing Piles. Rudolph Hering.
IV. The proposed removal of Smith's Island. Lewis M. Haupt.
V. Water-Supply for a Stamp Mill. By W. F. Biddle.
VI Empirical Formula for Strength of Wrought Iron Beams. P. Roberts, Jr.
VII. The Scales of Maps. Lewis M. Haupt.
VIII. The Strength of Wrought Iron in Structures. P. Roberts, Jr.

PROCEEDINGS OF THE BRITISH INSTITUTION OF CIVIL ENGINEERS.—The following Excerpt Minutes of the Proceedings have been, receivod through the kindness of Mr. James Forrest, A.I.C.E.:

Engineering Progress in Foreign Countries. By Messrs. Vernon Harcourt, Clark, Bauerman and Higgs.

Heating and Ventilating Apparatus of the Glasgow University. By W. W. Phipson, M.I.C.E.

Harbor and Dock Works. The Avenmouth Dock. John Bower Mackenzie, M.I.C.E.

The Harbor of Belfast. Thomas Ross Salmond, M.I.C.E.

Whitehaven Harbor and Dock. John Evelyn Williams, M.I.C.E.

Coal Washing Apparatus at Besseges, France. By J. B. Marsant.

LIVERPOOL ENGINEERING SOCIETY.—This Society held its usual fortnightly meeting at the Royal Institution on Wednesday evening last. Mr. M. E. Yeatman, M.A. President occupied the chair. A paper on "The Design and Construction of Sewers" was communicated by Mr. Graham Smith, past President In opening his subject, he remarked, that sewers should not only be constructed in such a manner to carry with dispatch to the outfall the sewage which might find its way into them, but they should likewise be built in a manner such that no portion of the sewage should percolate through them into the surrounding earth. Cholera for instance, might be spread indefinitely by matter discharged from the stomach or bowels of a cholera patient gaining access to any source of water supply. He quoted the General Board of Health returns to show that zymotic diseases largely exist and are much due to improperly constructed sewers and imperfect drainage arrangements. The question of the proper forms of sewers to be adopted under various circumstances was dealt with at some length, and the author advocated the circular section when a large and constant flow could be depended upon, and the egg-shaped sewer when a variable flow had to be accommodated, as the smallness of the invert increases the scouring action of a small quantity of sewage, whilst the increased size of the upper portion provides for any augmentation of flow. He considered that the success or non-success of any system of drainage depended upon the manner in which details were arranged. These questions and such others as the ventilation of sewers and the materials employed in their construction were fully considered.

IRON AND STEEL NOTES.

BRIGHT steel surfaces may be ornamented by painting the patterns intended to be left bright in Brunswick black. If the ornament is to be dead upon a bright ground the patterns must be left untouched, and the ground painted over. Aquafortis—diluted nitric acid—should then be poured upon the exposed parts of the steel, and in a few minutes it will be seen to have eaten sufficiently into the metal. Wash off the aquafortis with water. and Brunswick black may be removed with turpentine. If the steel is made blue by exposure to heat, the blue color can be removed, where it is not required, with white vinegar or other weak acid, the parts to remain blue being protected by Brunswick black. On the parts from which the blue is removed, further variety may be gained by painting additional ornaments in Brunswick black, and exposing the remainder of the ground to the action of the aquafortis. Gilding on steel was formerly performed with a spirit, but now it is best to send the work to an electro-gilder's, first painting over those parts not to be gilt with Brunswick black. The gilding may be performed at home by the following method:—It is known that if sulphuric ether and nitro muriate of gold are mixed together, the ether will, by degrees, separate from the acid nearly the whole of the gold, and retain it for a long time in solution. Take ether thus charged and with a soft brush paint the parts of the design intended to be in gold, and after giving the ether time to evaporate, rub over the parts thus gilt with a burnisher.

RAILWAY NOTES.

IN Europe it is common to indicate on posts the gradients of different sections of a railway at the points of change, for the benefit of the enginemen and others. Formerly on the Continent it was almost universally the custom to express the gradients in the form of a vulgar fraction, as is still done in England. Thus a grade of 1 in 100 would be indicated on the post by $\frac{1}{100}$. A short time ago this was changed on some Prussian lines to the form of a decimal fraction, as 0.01 for the grade above mentioned. The Prussian Ministry of Trade has now given instructions that this new method be abandoned and the old one substituted, the reason given being that the decimal fractions are often misunderstood by the enginemen.

PAPER AND STEEL *v.* CAST IRON RAILWAY WHEELS.—Much interest was excited in the railway world sometime since by the introduction in America of paper instead of cast iron wheels. There is no more important department of a railway than the rolling stock, and no part of the rolling stock of greater consequence than the running gear of trains. The paper wheel body is an American invention, of which Messrs. John Brown & Co., Atlas Works, Sheffield, are the makers for England. The paper wheel carries a steel tire, and it is claimed that the latter tire being truly turned,

and the two wheels on an axle being easily fitted to the same size, the pair will run much more quietly and easily than cast iron wheels of ordinary American make. Practical working, however, is better evidence than any quantity of "claims," and the *Chicago Railway Review* gives some interesting details of the records of the mileage made by the paper wheels as adopted by the Pullman Palace Car Company. The mileage of each trip is computed within one quarter of a mile, and the company, which has used the paper wheel with steel tire about nine years, knows exactly what its wheels are doing. The paper wheel costs £13 and runs 450,000 miles in 2.88 years. For convenience of reckoning call this period three years. At the end of three years the original cost with 7 per cent. compound interest, amounts to not quite £16. During this period nine cast iron wheels have been used, costing £2 16s., allowing a rebate of £1 each for the worn out wheels, and calculating on simple interest at 7 per cent., the cost of the wheels for this service amounts to £18 6s., showing a saving in the case of paper wheels of £2 6s., and with compound interest considerably more. In computing the cost for the second period of three years, a much greater saving would be shown, since a renewal of the tire only, at a cost of £7, is necessary, instead of a fresh cost of £13 for a new paper wheel. The experience of other railways bears out the records of the Pullman Company. The paper steel-tired wheel is used on the Central Vermont, Connecticut River, Cleveland, Columbus, Cincinnati and Indianopolis, Pittsburg, Cincinnati and St. Louis, and the Chicago and Alton line. Engine truck wheels have been found to run 800,000 miles before the tire requires renewal. The subject seems to be one which scarcely receives the attention it deserves in this country. We have as yet no data upon the relative durability of paper wheels, and those with wrought iron or wood bodies as commonly used ih England.

RAILWAYS IN CHINA.—When it was first understood that the Woosung and Shanghai Railway had been purchased by the Chinese Government, it was thought railways had obtained a footing in China. The news of its removal almost wholly as so much old iron, and that as the easiest means of transport the carriages were pitched into the sea to convey them to Formosa, the iron work tossed into barges, and the engines, partly in pieces, put into the cases in which they were sent to China in, and then the whole thrown on to the beach Formosa, soon, however, dispelled this idea, and hope of work for railway engineers in this field died away. It seems that the railway was laid while the governorship of Kiang Su was in the hands of a friend to progress, and who made no objections. When, however, the matter of the murder of Mr. Margery had to be settled, it was urged by Chinamen, who did not know much of Western ideas, that as a Chinaman had been killed on the Woosung railway, the work of the English, they had nothing to complain of. The only way of removing this notion was to make the railway Chinese property, and Li-Hung-Chang ordered the Governor of Kiang Su to buy the railway with whatever funds were at his disposal. In the meantime a new governor had been appointed who was not very well pleased with the order to buy the line, and he seems to have made up his mind that though he must buy it, the Minister of Public Works had no power to make him work it. Ting-Futai, the Governor of Formosa, was however, desirous of having a railway in his island, and wrote to the Governor of Kiang Su to that effect. The governor, in reply, said he had a railway he did not want and he accordingly pitchforked it over to Formosa in the way described The Woosung Railway had been at work just long enough to allow Chinamen to see what could be done with it, and that having had it they were inconvenienced without it. The result is that railways have been very much talked about since the removal, and have now many friends. Li Hung-Chang, one of the most advanced statesmen in China, has long advocated them, though he has worked under a disadvantage. Almost all the statesmen of China are Tartars; very few are Chinese, and the latter, when they do rise to positions, rise with great difficulty. The loss of the Woosung Railway has had the effect of stirring up interest in the matter, and with a good reason for believing that China will before very long accept the great civilizer, a well known English engineer has gone to China to negotiate for new lines, when the Celestial makes up his mind to admit them. It is also reported that Li Hung-Chang has contracted with Mr. Arnold Hague, a geologist and mining expert, for the purpose of prospecting for minerals in North of China.

A STRIKING illustration of the superiority of railways to the old coaches and diligence in point of safety has been published by M. Gorilaux, a French statistician. Before the introduction of railways, he estimates that one passenger was killed out of every 300,000 persons traveling by coach or diligence; while in addition there was one injured out of every 30,000 travelers. In the first twenty years of railway traveling in France, that is to say, from 1835 to 1855, the average number of passengers killed in railway accidents was one in every 2,000,000, and one injured in every 300,000. In the next twenty years, from 1855 to 1775. the safety of railway traveling had so much improved, that the average number of killed in accidents on the French lines amounted to only one person in every 6.000,000 passengers, while the injured numbered but one in every 600,000. Hence in the first twenty years, a railway journey was from seven to ten times as safe as a journey by the diligence. In the coach, a passenger ran seven times the risk of being killed, and ten times the risk of being injured that he did on the railway. In the next twenty years, says Herepath, he was twenty times more secure against the risk of a violent death or of injury, than if he used the lumbering old diligence, which, moreover, took five or six times as much of his time to perform a given journey. At the present time, M. Gorilaux estimates that the real danger to the life or limb of passengers from causes beyond their own control is some-

thing far less considerable than those we have just quoted. He calculates that on French railways, there are now actually killed only one person in every 45 000,000 passengers and one injured in every 1,000,000. On these results M. Gorilaux bases a computation by which he arrives at the striking fact in France that a passenger would have to travel at the rate of forty miles an hour, ten hours a day, without intermission for a period of 7,439 years, before he would have to fear an accident of a fatal character.

ENGINEERING STRUCTURES.

THE piercing of the St. Gothard tunnel advanced 21.7 meters on the Goeschenen side, and 30.1 meters from the Airolo side, or 52.1 meters during the week ending the 26th ult. During the week ending the 1st inst., the progress was 24.3 and 10.2 meters respectively at the Goschenen and Ariolo sides, or 35.2 meters. The boring now extends to 6421.3 meters on the Goschenen side, and 5906.9 on the Airolo side, leaving 2571.8 meters to be pierced in the center of the mountain.

THE CALAIS-DOUVRES.—From the last report of the London, Chatham, and Dover Railway Company we learn that the running of the twin Channel steamer Calais Douvres during the past season appears in most respects to have justified expectatation. She performed the trip from Dover to Calais and back for seventy-eight days with great regularity, making good and quick passages, and affording the public a large increase of comfort, and a very material diminution of those peculiar evils and annoyances heretofore incident to the sea transit. The general appreciation of her merits is shown in the fact that during the season upwards of 55,000 passengers crossed in her, or an average of 715 per day, a number which would on many occasions have involved the necessity of running two of the ordinary mail boats. This may be regarded as in some measure a set-off against the unquestionably heavy cost of work ing a ship of the size and capacity of the Calais-Douvres.

THE ST. GOTHARD TUNNEL.—M. Colladon recently gave some interesting particulars respecting this work in a paper communicated to the Academie des Sciences by M. Tresca. It seems that besides the excessive hardness of the beds of serpentine and quartz, and insufficiennt hydraulic power on the Airolo side, owing to the lowness of the water last winter, there has been a very heavy infiltration in the south portion, amounting to over 3,000 gallons per minute in the advance gallery. The jets of water had often the force of those from a fire engine pump, due to great heads. Another difficulty has been caused by a mass of decomposed felspar mixed with gypsum found under the plain of Andermatt, under the site of an ancient lake, and about 190 yards from the entrance. This plastic material swells on contact with moist air with irresistible force, capable of crushing the strongest timber, and even arches of granite 3.28 ft. thick. In some of these parts boring by hand advances about 1 metre in three or four days, while through granite, with compressed air and mechanical perforation, a regular advance of 10 ft. to 13 ft. in twenty-four hours has been achieved, and through gneiss as much as 20 ft. The volume of water from the Tremola—Airolo side—having been found insufficient, M. Favre brought water in an aqueduct 3.270 ft. long from the Tessin to work three new turbines, 16.4 ft. diameter, making 50 to 60 revolutions under a head of 262 ft., and four compressors. The old bronze turbines, 3.87 ft. diameter, working under a head 595 ft., which have made some 155,000,000 revolutions per annum, are in good preservation, after four or five years' service, and still work usefully. On each side of the tunnel there are, at present, sixteen air compressors in action, serving both for aeration and for boring operations. They send into the tunnel air under a pressure of eight atmospheres, sufficient to drive eighteen to twenty drills, making 150 to 160 strokes per minute, and effect good ventilation of the part already bored, which is at present 6,800 yards on the north side and 5,900 yards on the south. The transport of materials is effected by horses in the more advanced part of the tunnel, and by compressed air locomotives in the portions near the mouths. The Compagnie du Gothard, which has possession of the line, with the exception of the tunnel, has suspended work for two years. The excess of its actual over its first estimated expenditure is put at nearly 100,000,000f., principally owing to errors described in our corresponding article last year. The works of the tunnel have not been interrupted a single day for six years, and its cost, notwithstanding unforeseen difficulties, will exceed the estimates little, if at all.

AN INLAND AFRICAN SEA.—M. Ferdinand de Lesseps has read a paper to the French Academy of Sciences with reference to the interior African sea proposed by Captain Boudaire. M. de Lesseps sailed from Marseilles in November, 1878, and landed in the Bay of Gabes, into which flows the little river Melah, selected by Captain Boudaire as the channel communication between the Mediterranean Sea and the Chotts. Up this small stream the sea penetrates to a distance of about five miles at high tide. The last expression will naturally attract attention as the general belief has always hitherto been that no tide existed in the Mediterranean, with the exception of the Gulf of Venice, where a rise and fall of about 2 ft. occur. Such, however, is not the case, for at Gabes the tide rises to 8 ft. and sometimes 10 ft. The Tunisian Chotts commence at about 14 miles from the sea and extend to a distance of 250 miles. The ground which separates them from the sea would be easily worked. M. de Lesseps states that he noticed on the banks of the Melah some cliffs which appeared to consist of hard stone. He sent one of his companions to procure some specimens, and that gentleman broke off some pieces, which he placed in a bag in his pocket. When they came to be examined they were found to have all been reduced to small fragments, in fact, they consisted of nothing but agglutinized sand. A

portion of the material was handed to M. Daubrée for analysis, and he will make a report to the Academy on the subject. M. de Lesseps states that all the local traditions in Tunis agree as to the former existence there of an inland sea, and his opinion is that its re-establishment would be very easy. However, he said that nothing could be done until Captain Boudaire's mission of levelling and sounding was accomplished, and that would occupy at least six months. In reply to an observation by M. Cosson, M. de Lesseps stated that the formation of an inland sea would not interfere with the existing oases, as they are 15, 20, or even 30 meters above the proposed water level.

ORDNANCE AND NAVAL.

DIFFICULTIES are again experienced with the special 6 in. rockets ordered for the Afghan expedition. They have passed an unsatisfactory proof, two out of three having prematurely burst their cases during flight. Weighing nearly 100 lbs., which is four times the weight of the largest ordinary rocket, their cases were made of proportionate strength, wrought iron ¼ in. in thickness being employed. Based on the practice of the smaller missiles, this was calculated to afford ample resistance to the explosive gases escaping in rear; but the calculation has been proved to be incorrect and the strength of the iron cases insufficient, and they have been ripped open as if they were made of brown paper. The consequence is a dilemma, for the thickness of the cases cannot be much increased without serious detriment to the range and efficiency of the rocket, and it is evident that the cases as at present constructed are quite unserviceable. Fortunately, only 120 were ordered—the odd twenty being for experimental purposes—and they are not nearly completed. The probability is that they will be abandoned, at least for the present, as impracticable, and that the army in India will make the best of the 9-pounder and 24-pounder rockets, which, so far as rockets have any merit, did fairly well in Abyssinia and Ashantee.

THE AUSTRIAN GUNS OF THE UCHATIUS BRONZE STEEL.—A report upon the new Austrian 6 in. gun of bronze steel has been sent to the Royal Artillery Institution at Woolwich by Major J. F. Owen, Royal Artillery, who has for five years past been employed in the Royal Gun Factories, Royal Arsenal, Woolwich. He states that the field guns manufactured of the Uchatius metal in 1875 have given such satisfaction that the Austrians are now making on the same pattern much larger guns with a bore of 6 in. diameter. Although a breech-loader the 6 in. gun fires a charge of nearly 18 lbs. of powder with a shell weighing 85½ lbs., and has given velocities of 1,476 ft. at the muzzle, which is equivalent to a very satisfactory performance against ironclad ships. In accuracy and range the guns have also given good results; and as to endurance, Major Owen states that they have shown themselves as strong as the field guns. Notwithstanding considerable erosion of the bore the accuracy of fire had not diminished, and the powder chamber has remained intact. Altogether the report is favorable to the gun, but Major Owen remarks that although the adoption of the new metal by the Austrians may have been judicious for the time being on economical grounds, there being a number of old bronze guns on hand, and also because the manufacture could be speedily carried out in their arsenal, he does not suppose that in the long run the material will be able to hold its own against steel, in the production and manufacture of which very great progress has lately been made, as exemplified at the Paris Exhibition.

THE THUNDERER DISASTER.—A series of important experiments, suggested by the bursting of the gun on board the Thunderer, were carried out recently with the 38-ton guns on board the Dreadnought at Portsmouth. The experiments were conducted by Captain Herbert, of the gunnery ship Excellent, and among those present were Mr. George Rendel, the inventor of the hydraulic system of loading, Admiral Fanshawe, Rear-Admiral Foley, and Captain Lebrano, from the Italian Embassy. The purpose of the experiments was to ascertain whether the projectiles, after being rammed home by the hydraulic hammer at certain angles of depression, had a tendency to follow the rammers when withdrawn, and, secondly, when arrested by a wad, to determine the amount of force required to overcome the friction of the obstruction. A common shell was first tried without either a wad or a gas-check when it was found to slip back readily. A similar shell was then tried without a wad, but with a gas check affixed, with the same result. In both these instances the gun was depressed 14°, which is rather more than is requisite to load the Thunderer's guns. A common shell was next inserted in the gun, with gas-check and wad. It did not start, and it required the efforts of six men to pull it out, after three trials. A wad was also placed a few feet in the gun, without a shell, when it was found that it required eight men to pull it back. A new wad, suggested by Mr. Rendel, was afterwards tested. It is formed of lamina of different thicknesses, stitched together. A wad of single thickness took the strength of three men to remove; a wad of two thicknesses took four men to pull it out; while a wad of three lamina required five men to overcome it. The gun was afterwards charged with a common shell without wad. At seven degrees of inclination it slipped forward, and when the angle of depression was increased to nine degrees it ran forward 6 feet 3 inches. A dummy projectile was next tried, having a contrivance fitted to its base, the object of which was to enable the shot, should it slip forward, to draw the cartridge with it, and thus prevent the gun, in the circumstances, from going off. In two instances, the dummy did not take hold of the charge on being extracted, but in the third it brought out the cartridge. The last experiment was highly important. The gun was loaded with a Palliser shot, by means of the hand gear, without either wad or gas-check, but from a

horizontal position; The slide was then lifted to the upper step and the gun depressed and struck heavily on the port sill, but without starting the shot. The result of the experiment so far goes to prove that, without a wad, the projectiles will slip at seven degrees and over, but that with a wad the shot runs no chance of starting at the angle used for loading.

The gunnery experiments on board the Dreadnought, sister ship of the Thunderer, were concluded at Portsmouth on Wednesday. Previous to resuming them Captain Herbert and Mr. George Rendel paid a visit to the Inflexible, which is being fitted with the latter's hydraulic gear. But here a great improvement has been introduced, not, however, as regards the principle of the hydraulic system, but in respect to the construction of the ship itself for its application. The upper deck in wake of the turrets has an undulatory appearance, so that the guns may be rotated under the cover of the highest part to load. By these means the 80-ton guns will be loaded from below, at the reduced angle of 8°, instead of 14° on board the Dreadnought, or 11½° on board the Thunderer. The experiments of Wednesday were of a very different character from those of the preceding day, and were undertaken for the purpose of determining the truth or fallacy of certain theories which had been put forward, and more particularly by several correspondents in the *Times*. It required no elaborate experiments to prove that a shot had a natural tendency to slip back when the muzzle of the gun was depressed; and that at an angle of 11° or 14° of inclination it would certainly do so by a wad, or by the foulness of the bore when it frequently holds, especially with a gas-check leaving little windage. But it was alleged that the wads used on board the Thunderer and the Dreadnought were not a sufficient protection against the starting of the shot, and that this starting might be assisted down the incline, first, by the pressure of the condensed air at the rear of the shot after being rammed home, and which had no means of escape; secondly, by the suction or vacuum produced in front of the projectile by the rammer on being withdrawn, and which imparted a tendency of the shot to follow the rammer; and, lastly, by the united force of these two disturbing influences. Arguments of this kind could only have been put forward by persons practially unacquainted with the hydraulic system of loading, and more particularly with the character of the rammer head. Nevertheless, as such opinions had gained currency, it was resolved to test their value by experiment, and the trials on board the Dreadnought, were for the purpose of ascertaining the force of the alleged air-pressure behind the shot and the alleged suction between the wad and the rammer head, to push the projectile forward. For this purpose the ventilating holes in the rammer were plugged up—as it seems there are no apertures in the rammers used on board the Thunderer—and a mercurial pressure gauge was attached to the vent of the gun and a second one to the rammer head. As the shot was forced rapidly down the bore of the gun for half its distance by the first length of the rammer the mercury rose an inch in the tube, indicating a pressure of ¼ lb. per square inch of the shot's area—a perfectly insignificant amount; but before the second half of the telescopic rammer (which works less rapidly than the first) could force the shot home upon its seat even this pressure had vanished, the air escaping through the grooves of the rifling and the interstices of the sponge. The charging was next tried with a view of testing the vacuum. Here, again, the result was found to be perfectly satisfactory and reassuring, the vacuum being quite harmless. But the most unexpected result of the experiment was the discovery that the vacuum which was found to be produced in rear of the projectile, and the tendency of which was to pull it back, was only to an infinitesimal degree less appreciable than the vacuum found in front, and which had a tendency to pull it forward. The experiments thus far have proved eminently satisfactory so far as the system adopted for loading is concerned, and go far to show that the Thunderer's gun exploded through the starting of the shot—which remains to be proved, and which will probably have to be ascertained by experiment—the slipping back can only be attributed to the omission of the wad during the rapid practice. In describing the last and most important experiment of Tuesday it was stated that the round was loaded without either wad or gas check and while the gun was in a horizontal position. This was an error. Both on board the Thunderer and the sister ship, the 38-ton guns are loaded by hand at precisely the same depression as is required by the hydraulic apparatus, the advantage of horizontal loading having had to be sacrificed to the necessity of adding 3 feet to the gun. Both wad and gas-check were used on the occasion, and the object of the experiment was to ascertain if a wad capable of holding the shot and charge securely from any risk of slipping forward could be put up by the hand rammer which has been provided to replace the hydraulic rammer in the contingency of the latter failing. The hydraulic rammer was accordingly removed and the alternative gear employed, but in the same position; and not only did the wad hold so well that the shot could not be started by any inclination or jerking of the gun, but it required the combined efforts of eight men to remove it. The supreme purpose of the experiments has been to ascertain the precise amount of the tendency of the shot to run forward, and to compare it with the amount of hold of the wad provided to prevent the slipping. At 14° the force of the shot to move forward is 1 in 4, or, allowing for friction, the tendency does not exceed 80 lbs. even in a clean gun, while, on the other hand, the wads provided for the ship are found to require over half a ton and upwards to pull them out. At 8° the tendency is 1 in 7, or 100 lbs., but, deducting the same allowance for friction, the result is an equilibrium of force. It may be worth stating that wads of some sort are required for all guns, even on the broadside.

The experiments at the Royal Gun Factories in the Royal Arsenal, Woolwich, to throw light upon the causes of the explosion of the 38-ton gun on board Her Majesty's ironclad

turret ship Thunderer were completed on Tuesday morning, and the tendency of projectiles to slip forward when the muzzles of the guns are depressed has been tested with each description of heavy gun in the service. In every case the shot slipped, sometimes at a depression of 8°, and always before reaching the angle of loading by the hydraulic machinery, which is about 10 or 12°. The bore of the 38 ton guns is 16 feet 6 inches in length, and it is the opinion of the authorities at the Royal Gun Factories that the shell in the Thunderer's gun had slipped to a position about one-fourth of the way, which would correspond externally with the front of the trunnions. Major Owen, Royal Artillery, the representative of the War Office, has arrived at Malta, but has not yet communicated his conclusions to the authorities at home. It is understood that, notwithstanding the frightful nature of the shock caused by the explosion, the turret of the Thunderer still revolves, and that the fellow gun in the turret could in a very short time have been worked and fought had it been necessary to do so. The committee appointed by the Admiralty to inquire into the recent disaster consists chiefly of naval officers now at Malta, Mr. Bramwell, F. R.S., acting as assessor. Captain Noble, late of the Royal Artillery, will attend to represent Sir Willliam Armstrong & Co., the manufacturers of the hydraulic machinery for loading and working the guns in the fore turret on board the Thunderer.

BOOK NOTICES.

TURBINE WHEELS. By PROF. W. P. TROWBRIDGE, of Columbia College. Van Nostrand's Science Series No. 44. New York: D. Van Nostrand. Price 50 cts.

The object of this essay is to show that the theory of Turbines as set forth by Rankine, Weisbach, Bresse, and others, is not a satisfactory elucidation of the principles upon which the modern turbines are based.

The author says: "It may be shown, I think, that if Boyden and Francis had followed strictly the rules of construction laid down in the works alluded to, they would have failed in their efforts to construct Turbines giving any considerable increase of efficiency over the old Fourneyron or Journal wheels of European design or construction."

The discussion of the essay is intended to apply to the later forms of Turbines, but the formulas are applicable to nearly all water wheels, by making simple and proper suppositions in regard to the quantities which enter into them.

MANUAL OF PRACTICAL CHEMISTRY. THE ANALYSIS OF FOODS AND THE DETECTION OF POISONS. By ALEXANDER BLYTH, F.C.S. London: Charles Griffin & Co. For sale by D. Van Nostrand. Price $5.00.

This work does not present, in any part, the ordinary scheme for qualitative analysis. To employ the book to advantage, the reader must be familiar with the methods and materials of a laboratory.

The treatise is in two divisions, land is further subdivided into "parts" as follows:

First Division. Analysis of Foods.

Part I, Ash, Sugar, Starches; II, Wheat, Flour, Bread; III, Milk, Butter, Cheese; IV, Tea, Coffee, Cocoa; V, Alcohol, Alcoholic Liquors; VI, Vinegar; VII, Mustard, Pepper, Almonds, Annatto.

Second Division.

Part I, Introductory; II, Prussic Acid, Phosphorus, Chloroform; III, Alkaloids and Vegetable separated for the most part by Alcohol; IV, Animal Poisons; V, Inorganic Poisons.

In the first division will be found most of the valuable information that is not obtainable from other sources, and in this portion the author has presented a large collection of analysis of the common things of the household.

INSTRUCTIONS FOR TESTING TELEGRAPH LINES AND THE TECHNICAL ARRANGEMENT OF OFFICES; WRITTEN IN BEHALF OF THE GOVERNMENT OF INDIA, UNDER THE ORDERS OF THE DIRECTOR GENERAL OF TELEGRAPHS IN INDIA. By LOUIS SCHWENDLER. Vol. I. London: Trübner & Co. New York: D. Van Nostrand. 1879. Price $4.00.

The first volume of this work, which has recently been issued, is in substance a revised compilation of the contents of a series of circulars of instruction relating to the practical work of the Government telegraph lines in India, which were prepared by Mr. Schwendler at intervals from 1869 to 1876, and printed for the especial use of the staff in that department. The very general demand among members of the profession in other countries for copies of these circulars, finally led Mr. Schwendler to undertake the preparation of a second and revised edition in a more convenient and accessible form. The instructions in the present work, although thus prepared for a special purpose and under the influence of local circumstances, are almost equally well adapted to the use of practical electricians in other countries. In fact, it may be said, that the peculiar difficulties which are met with in the construction, maintenance and administration of telegraphic lines in India, will be found to exist to a much greater extent on the American continent than in any part of Europe, and therefore it is not unlikely that Mr. Schwendler's work will ultimately prove of more real value in this country than anywhere else outside of India.

The present volume is devoted entirely to the subject of testing; the portion relating to the technical arrangement of offices, etc., being reserved for the succeeding volume, which is intended to be published some time in the course of the present year.

The first volume is divided into two parts. The first part treats of the apparatus employed in line testing, *viz.*, the so-called "Wheatstone" bridge, and the differential galvanometer. Of these two methods, Mr. Schwendler very properly gives the most decided peference to the former. The differential galvanometer, he says, for the same bulk and cost, can never combine the same accuracy and sensibility within wide limits as the bridge.

The mathematical theory of the bridge under various conditions is given at great length, and with an imposing array of formulæ. A theoretical and an actual plan of a testing board is given at the end of this section, which embodies several very valuable features not often found, if at all, in the ordinary arrangements familiar to electricians. For example, the galvanometer coil is wound in two sections and provided with a commutator, which enables the sections to be placed in series or in multiple arc, according as the resistance to be measured is great or small. Another commutator provides for interchanging the position of the battery and galvanometer in their respective diagonals, a great convenience when very large and very small resistances are required to be measured by the same apparatus.

The second part is devoted solely to the examination and solution of the problems which practically arise in the electrical testing of telegraph lines; the determination of their general electrical condition, the localization of faults, etc., etc. Although this chapter is very full, and exceedingly well arranged, there is, perhaps, little or nothing contained in it which is new to the well-informed electrician. An unusual degree of attention is devoted to the methods of deducing the true electrical condition of the line from the apparent condition as shown by the results of the actual tests, a matter which has not by any means received due attention in most of the existing test-books. The directions for checking and eliminating untrustworthy measurements, in making up averages, are also of great value, since this is a matter that frequently leads to error on the part of inexperienced practitioners. So far as we have been able to discover, almost every condition likely to come up in practice has been duly provided for, and this, by the way, is an advantage which has resulted from the manner in which the book has grown up, piecemeal, as it were, in accordance with the requirements of actual service, as manifested from time to time.

Not the least useful part of the work are the various appendices, giving actual examples, taken from the records of the Indian lines, which show the practical application of the methods set forth, in detail. Specimens of all the blank forms, for records, reports, etc., which are there used, are given, and altogether this part of the work will hereafter prove of great service to the officers of other administrations, who are desirous of organizing a system of testing for the purpose of securing similar results.

We think it would have added very much to the practical value of Mr. Schwendler's work, if he had devoted a reasonable amount of space to the theory and practice of line testing with the tangent galvanometer, which, although not to be compared with the differential apparatus in accuracy of its results, is nevertheless a most admirable instrument for ordinary every-day work, in the hands of the average operator. By means of it, it is easy to keep the run of the condition of the lines by going over them rapidly every morning, and in case any fault is discovered, the more accurate differential methods may be resorted to for quantitive measurements, as is now the general practice in this country. The excellent and convenient instruments constructed by Mr. Phelps have proved very serviceable for this work, and have received frequent approval from foreign electricians who have seen them in use here. It is not very probable that so well-informed and experienced an electrician as Mr. Schwendler, is altogether ignorant of the value of the tangent galvanometer when used in this way, and we are, therefore, disposed to find a more probable explanation in the fact that the conditions of the Indian service are unlike ours in one important respect, viz., the much greater average length of circuits and distances between testing stations, and the much smaller number of wires in each office. This renders a more considerable degree of accuracy necessary, and at the same time enables it to be obtained with less inconvenience. Mr. Schwendler's precise habits of mind may also have led him to undervalue the "rough and ready" processes and loose approximations of this mode of measurement. He is a skillful mathematician, as the pages of the work under consideration sufficiently attest; and doubtless has the true mathematician's horror of inexact results. Nevertheless, it is to be hoped he will in a future edition take up this matter, and handle it with his accustomed skill.

The typographical execution of the work is excellent The type is clear and open, and the paper and press work unexceptionable The succeeding volume, treating, as it will, upon a branch of telegraphic engineering which has been almost entirely overlooked not alone by the writers of test-books, but even by the contributors of the professional journal, cannot fail to be of great practical value. Its appearance will be awaited with interest, especially in this country, where far greater attention appears to have been paid to this subject than has been the case abroad.—*Journal of The Telegraph.*

ARMY SACRIFICES; OR BRIEFS FROM OFFICIAL PIGEON-HOLES. By JAMES B. FRY, Colonel Adjutant General's Department, and Brevet Major-General United States Army. New York: D. Van Nostrand. 1879. Price $1.00.

This is a neat volume of sketches which have been "grouped together for the purpose of illustrating the services and experiences of the regular Army of the United States on the Indian frontier." Notwithstanding our innumerable Indian fights, there are very many Americans who have no correct idea of the real life of a soldier in the far West. Here, in the East, officers and soldiers have a comparatively easy time of it; but the ordinary duties in the West are not only without glory, but are attended with dangers and sufferings no less terrible and trying than those of the great wars whose victims receive the unstinted sympathy of their fellow-men, and whose heroes are held up forever to the admiring gaze of millions. The savage foe, the sudden storm, the pitiless wind, the biting frost, the frequent hunger, encountered every year by the larger part of the

Army, call for an aggregate of courage, skill, and fortitude which is perhaps not found elsewhere in the world. The book before us is a recital of some of the more thrilling incidents which have occured from time to time west of the Mississippi River. Most of the sketches are of actual occurrences, and are presented as examples of the dangers and privations to which our soldiers in active service in the Indian country are continually exposed and the gallantry and fortitude they display. So far as these are concerned, the book is a creditable reproduction of official reports, elaborated, perhaps, by confirmatory information of a reliable character. Some of them would have been improved by the elimination of certain introductory remarks at the beginning of each new piece, and the striking out of the too frequent notification that "the first effort of Indians in making an attack is almost invariably to stampede the animals." Nor can we approve of two or three selections, which, to our mind, have no connection whatever with Army life, and which we cannot believe form any part of official records. For instance, the description of the "Penitentes," a religious sect of New-Mexico, while highly interesting in its hideous details, is certainly out of place in a book illustrative of Army sacrifices; as is also the story of "Bill and Dan," two industrious brothers, who, unfortunately for them, no doubt, "met a tragic fate," but one not more tragic than that suffered by hundreds of other frontiersmen.

There are soldiers who fell in the cheerful performance of arduous and dangerous duties whose deeds and death are almost forgotten, save by their families and a few frontiersmen who, in their wanderings, may cross the trails or streams which bear their names. Fortunately, this book contains very many examples of this sort, records of unsurpassed bravery and endurance, of the deeds of men who were not afraid of death, and who did their whole duty as best becomes a soldier. Fiction itself can present no more pathetic incidents or more stirring tales of heroism than some which are so graphically presented to us by General Fry. The story of Lieut. Mellen's sufferings—the summons to duty at a distant post, the lonely ride, the swollen river, the noble horse and gallant rider breasting the turbid waters, the sudden slip and struggle amid the icy waves, the grasp for life, the long dead blank, the painful awakening to a possible death from cold and starvation, the terrible efforts to regain the saddle, the agonizing ride, and the final amputation of both feet—all these were but actual occurrences, growing out of the execution of a simple order in the ordinary routine of an officer's life on the frontier. Take, also, the case of Brevet Lieut-Col. Forsyth, of which we shall have more to say further on, and of Major-Gen. George L. Hartsuff, a hero of the civil war, who died in this city, in 1874. This gallant soldier's fight, when but a Lieutenant, with a band of Seminoles in Florida, in December, 1855, is a wonderful story. The description of Roman Nose, a great war leader, is worthy of quotation. "He was," says Gen. Fry, "one of the finest specimens of the untamed savage. It would be difficult to exaggerate in describing his superb physique. A veritable man of war, the shock of battle and scenes of carnage and cruelty were as the breath of his nostrils; about 30 years of age, standing six feet three inches in height, he towered, giant-like, above his companions. A grand head, with strongly marked features, lighted by a pair of fierce black eyes, a large mouth with thin lips, through which gleamed rows of strong white teeth, a Roman nose, with dilated nostrils like those of a thoroughbred horse, first attracted attention, while a broad chest, with symmetrical limbs on which the muscles under the bronze of his skin stood out like twisted wire, are some of the points of this splendid animal. Clad in buckskin leggings and moccasins elaborately embroidered with beads and feathers, with a single eagle feather in his scalp-lock, and with that rarest of robes, a white buffalo, beautifully tanned and soft as cashmere, thrown over his naked shoulders, he stood forth the war chief of the Cheyennes." The occasion was a council at Fort Ellsworth, late in 1866, at which the Indians protested against the building of railroads, because they were driving away the buffalo, and thus interfering with their hunting grounds.

Roman Nose was afterward killed while leading a large band of savages against Major George A. Forsyth, of the Ninth Cavalry. Forsyth was then, as he is now, on Gen. Sheridan's staff. He had accompanied his chief to the far West, and chafing under the restraint and inactivity of his position, begged a command in the field. It was finally decided that he might lead a force of 50 men, who had been specially hired for the occasion. This little force was soon in the saddle and in active pursuit of the Indians. On the night of the 16th of September, 1867, Major Forsyth's force was surrounded by nearly a thousand savages, and then followed one of the most heroic and effectual struggles in the history of mankind. The Indians rushed in upon the heroic half a hundred time and time again, but as often were they received by the well-directed fire of musketry. At the close of the first day, every horse and mule was dead, Lieut. Beecher, the second in command, was killed, three men, including the doctor, lay dead in the "trenches," two others were mortally, and 17 severely wounded. Before 10 o'clock, Forsyth himself had been shot in the thigh; a few hours afterward a ball entered his left leg below the knee, completely shattering the bone, and before night a third ball grazed the top of his forehead, chipping out a piece of the skull. Men volunteered to go out in search of relief. Day after day the fight was renewed and all the food our heroes had was raw mule meat. But presently, it grew putrid, could no longer be eaten, and the pangs of hunger began. By the seventh day the Indians had entirely disappeared, but the beleaguered force were now to weak to move. Men became delirious, and the wounded ones suffered dreadfully. The shattered bone of brave Forsyth's leg stuck through the skin, and

maggots took possession of the horrible sore. The eighth day wore away, and then, on the morning of the ninth day, friends arrived. The empty honor of a "brevet" was the only recognition Forsyth received for his heroic conduct in this affair, and well may Gen. Fry ask, "Can bravery, gallantry, and devotion to duty flourish under a military system in which such services are neither rewarded nor remembered?"

In his preface, Gen. Fry says it is unavoidable that the recitals in those pages exhibit the worst features of the Indian character, such not being the purpose of the work. "Driven continually behind our rapidly-advancing frontier, plundered and abused by the more powerful and aggressive race, without one particle of redress for any wrong done him by the white man, and knowing no law but that of retaliation and vengeance, it is not strange that the barbarian should indulge in bloody deeds." After discussing this question at some length, the General arrives at certain conclusions, which, briefly summarized, are: First, localize the Indians, subdividing tribes into bands, and securing the title of land to them in common by a deed of trust; secondly, place these locations under martial law, to be administered upon white men and Indians alike; and, thirdly, permit all proper intercourse, especially inter-marriage, between the whites and Indians.—*N. Y. Times.*

MISCELLANEOUS.

MR. GALLOWAY'S theory relating to the part played by floating coal-dust in mines, is receiving much support. M. L. Simonin recently stated certain facts which he considers prove that in the majority of cases it is to the heating of the coal-dust diffused in the galleries of the mines to which explosions are due. Referring to the catastrophe in the Jabin Mine, at St. Etienne, February 4th, 1876, he states, on the authority of the manager, that the mine in question contains very little fire-damp, and that the precautions hitherto taken, with an exclusive reference to that gas, are not sufficient Others must be taken against the extremely fine coal-dust, which at the moment of the explosion of slight amounts of fire-damp, or even of blasting powder, liberates rapidly a part of the coal-gas which it contains) and propagates the explosion, reproducing the cause of the evil with so much the greater energy as the current of air is more violent. Thick crusts of coke—2 or three centimeters—prove this fact, and explain how it is that extensive tracts in which fire-damp has never been observed are burnt like the rest of the workings. Hence, it appears that the precautions to be taken in fiery mines are complex whenever the coal-dust is rich in gas and very finely divided. Explosions may then ensue even in mines where fire-damp is unknown. There is no need to suggest the existence of cavities full of carbonic oxide, or of gaseous hydrocarbons, and suddenly laid open by a blow from a miner's pick. The *Chemical News* suggests that the attempt, recently made in some mines, to screen the coal below ground, is a dangerous mistake, as calculated to increase the quantity of dust diffused in the air of the mine.

TIRE FASTENINGS.—A new and valuable method of fastenings tires, invented by Mr. Kaselowsky, a German engineer, has successfully stood the test of a series of experiments. A dovetailed groove is turned in the inner face of the tire, and a similar one in the outside of the skeleton, so that, when the tire is slipped on, the two come opposite to each other and form a channel of dowel-shaped section going all round the wheel. Into this channel is run some easily fusible metal (by preference pure zinc), which, on cooling, makes a firm connection between the tire and wheel. In carrying out the operation the tire is only slightly heated, a shrinkage of $\frac{1}{1000}$ being found ample, and is then brought over the skeleton, which is laid in a horizontal position, and forced upon it. The zinc is then immediately run in through the holes cast in the skeleton, if of cast metal, or drilled in other cases; thus the zinc is at once prevented from cooling while being run in, and is compressed, and thus rendered much stronger, by the subsequent contraction of the tire. That this mode of fastening, in addition to its simplicity and cheapness, offers full security, both against sideways shifting, and in case of breakage of the tire, has been proved by experiments made in the central workshops at Frankfort.

IN a paper read before the French Association, in August, M. Daubree, director of the School of Mines of Paris, says that one of the most remarkable characters of the rocks which have undergone mineralogical transformations, comprised under the name of "metamorphism," is that the rocks thus transformed are often associated, occupying together considerable territory, while in other regions, the same rocks still more extensive, do not present like modifications. These transformations are probably due to the influence of heat produced by mechanical actions, that have left their traces in the bendings and foldings of the strata. M. Daubree, after a series of experiments on the heat produced in rocks by interior movements, draws the following conclusions: (1) The rocks were already in a solid state at the period when they followed the action which contorted them. (2) Many of these rocks during these movements acquired a laminated structure. (3) Certain effects of regional metamorphism may be derived simply from the heat which has been developed in the rocks by mechanical action. (4) Fossils have been destroyed by trituration in the interior movements of such rocks as have become changed in texture or assumed a crystalline state. "Finally," says M. Daubree, "in rock masses where metamorphism has been developed on a great scale, and far from any eruptive rock, the heat which has presided over the transformation of the rocks, and the appearance of new species of minerals, may have been caused by the very mechanical actions which these rocks underwent." M. Daubree thus seeks an explanation of the evidences of heat action in the mechanical action resulting from loss of heat, to which Mallet ascribed volcanic and earthquake phenomena.